Predicting the Energy of Finite Time RF Pulses.

P. Le Roux[1] and R. F. Schulte[2]

[1]Applied Science Lab, GE Healthcare, Palaiseau, France, [2]Imaging Technologies, GE Global Research, Munich, Germany

Introduction

The Shinnar-Le Roux (SLR) transformation (1) provides a semi-analytic way to solve the problem of RF pulse design. Another tool permitting design of RF pulses (although of infinite time support) is the Inverse Scattering Transform (IST). Recently, a formula was derived for the Inverse Scattering Transform (2) permitting the computation of the energy of RF pulses from its specification in the spinor domain. This formula is given without demonstration, relying on difficult mathematical results. It is also suggested in (2), but again without details, that a similar formula could be attained using the SLR approach. One objective of the current work was to derive this relationship using the SLR transformation and in a simple manner.

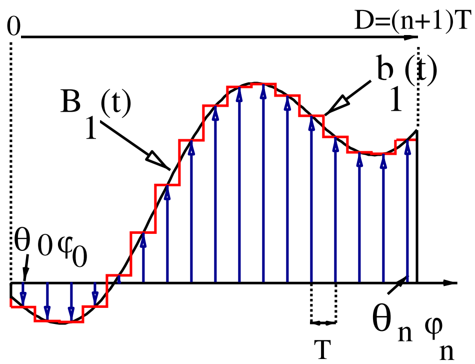

Figure 1: Depiction of a continuous excitation $B_1(t)$ modeled by an hard pulse train, as in SLR. The intermediary staircase signal $b_1(t)$ is useful for the analysis.

Method

Figure 1 recalls the 'hard pulse' approximation used by the SLR algorithm where a continuous envelope $B_1(t)$ of duration D is approximated by a train of n hard pulses. An intermediary approximation is the staircase signal $b_1(t)$, which has a time integral equal to the nutation of the corresponding hard pulse for each interval. An elementary calculation shows, that the energy of the signal $b_1(t)$ is proportional to the sum of squares of the nutation as expressed on the left of relation [1] in Fig. 2. Under most circumstances, and particularly if $B_1(t)$ is continuous, the staircase signal b_1 tends towards the continuous signal B_1 when n increases, and thus its energy E converges towards the energy of the continuous waveform when T decreases. Independently, an important result from the SLR algorithm is that the leading coefficient a_0 of the A polynomial is the product of the cosine of half the nutation angles. This is better expressed taking the logarithm and replacing the product by a sum as we have done on the left of the expression [2]. For decreasing T, the number n of hard pulses increases, but the log of each cosine decreases quadratically and finally the logarithm of a_0 tends toward the sum of square of the nutation divided by (-8). Putting together the two results, one can express the energy E in proportion of the -ln(a_0) as on the left of relation [3]. A classical result of discrete signal analysis (3,4) is the ability to express ln(a_0) of a minimum phase signal, the common choice for A (1), in terms of the mean value on the unit circle of the logarithm of its magnitude. This is the first term of the last member of relation [3]. Also, one may chose to take another realisation of the A by mirroring one or several zeroes z_l along the unit circle (5,6), as shown in Fig. 3. Each zero mirroring multiplies a_0 by the modulus of the mirrored zero $|z_l|$ and this leads to the second term in the last member of relation [3]. This readily permits to calculate the energy of the staircase signal $b_1(t)$, before the inverse SLR recursion and during the design of the A polynomial. When doing that in practice one will replace the integration on the unit circle by the summation of the discrete values after an FFT, dividing the result by the number of values *nfft*. This is feasible for a given value of T, but because T appears in the expression [3], one may wonder about the convergence of the energy E towards a constant when changing T. When performing several designs with T decreasing, one can observe a simple contraction of the response on the unit circle (Fig. 3), in accordance to the mapping on the unit circle expressed by [4], but also a convergence of the zeroes of the A polynomial towards the unit circle corresponding to the general mapping giving in [5]. Rewriting [3] with the aid of [4][5] gives the energy in terms of the Fourier transform of a continuous signal as in [6], where T disappears. This can be considered as the energy of the continuous excitation $B_1(t)$ towards which b_1(t) converges, and it is the strict equivalent of equation 50 in the reference (2), the discrepancy by a factor of two coming from the fact that all the radial frequencies has been divided by two in reference (2) compared to their accepted physical meaning.

$$\sum_{i=0}^{n}\theta_i^2=\gamma^2T\int_0^D|b_1(t)|^2\mathrm{d}t\xrightarrow{T\to 0}\gamma^2T\int_0^D|B_1(t)|^2\mathrm{d}t \qquad [1]$$

$$\ln(a_0)=\sum_{i=0}^{n}\ln(\cos(\theta_i/2))\xrightarrow{T\to 0}-\frac{1}{8}\sum_{i=0}^{n}\theta_i^2 \qquad [2]$$

$$E=-\frac{8}{\gamma^2T}\ln(a_0)=-\frac{8}{\gamma^2T}\left[\frac{1}{2\pi}\oint\ln|A(\varphi)|\,\mathrm{d}\varphi+\sum_l\ln|z_l|\right] \qquad [3]$$

$$\varphi=\omega T \quad [4] \quad , \qquad Z=e^{pT}=e^{(r+j\omega)T}\ , \quad \ln Z=Tp \qquad [5]$$

$$E=-\frac{8}{\gamma^2}\left[\frac{1}{2\pi}\int\ln|A(\omega)|\,\mathrm{d}\omega+\sum_l r_l\right] \qquad [6]$$

Figure 2: Mathematical relations used in text.

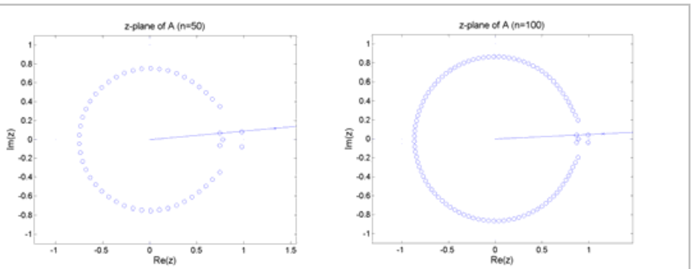

Figure 3: Complex z-plane position of the zeroes of the minimum phase A with n=50 (left) and n=100 (right). When reducing T there is a contraction of the response on the unit circle and corresponding movement of the zeros position with an increase of their radius, according to Eq [5].

Results

In practice, at least in he context of *SLR* design, one will use the discrete formula [3], pertaining to a particular Z plane . This easily permits to predict the energy cost of any zero mirroring. To verify the relationship, a pulse was designed with a time bandwidth of 5, Fig. 3. The zeroes of the A polynomial in the passband of the RF pulse were systematically flipped, leading to a ratio of the energy, obtained simply by summation of the $|b_1|^2$ waveform, over the values obtained by [3] of 0.996±0.002, verifying our derivation. In practise though, the blind use of the Remez algorithm would lead to spikes at both ends of the RF pulse, and the energy will increase when T decreases. To guarantee convergence of b_1 towards a continuous signal B_1 but also, more pragmatically, to redude energy and peak power, one must use an Interpolated FIR filter (IFIR) (7) procedure when designing the B polynomial. Then the results obtained by [3] stabilize, becoming independent of T as soon as this value of T is small enough.

Conclusion

A correct relation [6] giving the energy of RF pulse has been derived. In contrast to the *IST* derived equation (2), it is valid for finite time support excitations.

References

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