Non-iterative decomposition of fat and water using chemical shift

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INTRODUCTION

The symmetric sampling schemes used in classical Dixon fat-water decomposition schemes often led to redundant measurements and hence ambiguous estimates [1]. Since they were designed for specific patterns, their extension to alternate patterns and multiple metabolites are non-obvious. To eliminate these problems and to improve noise robustness, iterative schemes such as IDEAL were introduced [2]. While these schemes are flexible, the maximum likelihood criterion used by them has multiple local minima [5] (even when all the metabolites are present). When some of the metabolite concentrations are zero, there exist multiple feasible solutions; since the algorithm assumes a single solution, it converges to one of the feasible roots. Both of these issues make the approach very sensitive to initialization.

The main focus of this paper is a flexible and non-iterative algorithm for the decomposition of metabolites, which is applicable to arbitrary sampling patterns and any number of metabolites. We modified the standard harmonic retrieval (HR) framework [3] to account for the dependence of the frequencies to a single unknown parameter: the magnetic-field inhomogeneity. In contrast to the linear system of equations in standard HR formulation, we obtain a system of polynomial equations. Using algebraic techniques, we solve for the common root of these polynomials to estimate the inhomogeneity map. Thanks to the algebraic framework, there exist efficient numerical algorithms to estimate the number of feasible solutions and the field-map. In contrast to schemes that assume unique solutions, the proposed scheme provides a set of feasible solutions at every pixel; the set will have a unique root, when both fat and water are present. We then use the prior knowledge of the smoothness of the field-map to choose the correct roots from the feasible sets. We formulate the root selection as an integer programming problem, where one of the feasible solutions is chosen such that a specified smoothness criterion is minimized. Since this approach is remarkably similar to phase unwrapping, we solve the phase-unwrapping and root-selection problems in a single step, using a region merging algorithm.

METHOD

The signal model for N metabolites, with known chemical shifts and the unknown field inhomogeneity $\omega(x)$ is given by $s_n = s(x, nT+\delta) = \sum_{n=0}^{N-1} \rho_n' (e_n y)^n$. Here, $\rho_n' = \rho_n \exp(\mathrm{i}[\omega_n + \psi]\delta)$, $e_n = \exp(\mathrm{i}\omega_n T)$ and $y = \exp(\mathrm{i}\psi T)$. ρ_n are the metabolite concentrations and δ is an arbitrary shift in sample timing. Using the harmonic retrieval framework [3], we obtain the forward filter that annihilates the signal samples as $h(z) = \prod_{n=0}^{N-1} (1 - e_n y\, z^{-1})$. Since the coefficients of this filter are monomials in y, we obtain a polynomial equation in y when applied to the signal samples,. Similarly, we obtain another polynomial equation in y using the backward annihilating filter. The field inhomogeneity term can be evaluated as the common root of these two polynomial equations. We then use an algebraic framework using the Sylvester matrix of the polynomials (denoted by **S**) to derive the common root [4]. Thanks to the framework, the number of feasible solutions is efficiently estimated prior to the evaluation of the common roots. Aided by the estimate of the number of roots, the gcd evaluation algorithm provides a set of feasible solutions at each pixel; we use the prior information of the smoothness of the field map to identify the correct root from the feasible set. We formulate the selection problem as an integer programming scheme and solve it using the region merging algorithm. Briefly, our algorithm has the following steps: (a) **Model order estimation:** we estimate the order using a singular value decomposition of **S.** Aided by the estimated model order, we denoise **S** to **S'** using the corresponding low-rank approximation **(b) GCD evaluation**: involves the LU decomposition of **S'**. (c) **Ambiguity removal:** Use a region merging algorithm to minimize a global smoothness criterion, thus selecting the appropriate root at each pixel. (d) **Concentration estimation:** Given the inhomogeneity term, the concentrations are estimated using a least squares algorithm [2].

RESULTS

In this section, we validate the algorithm with an oil-water phantom and brain data. The images in Fig.1 were acquired on a 3T Siemens Allegra scanner using a modifed spin echo sequence (TR=1.6 s,TE=30ms). The readouts were shifted with respect to the center of the echo by 200, 600 and 1400 ms with the center of the echo. To test the algorithm on human data, we used the IDEAL sequence [2]; it consists of three gradient echoes (TR=0.5s) corresponding to TE=4.170,4.97 and 5.77 ms respectively. The decompositions using the proposed algorithm and the results are listed as in Fig. 3.

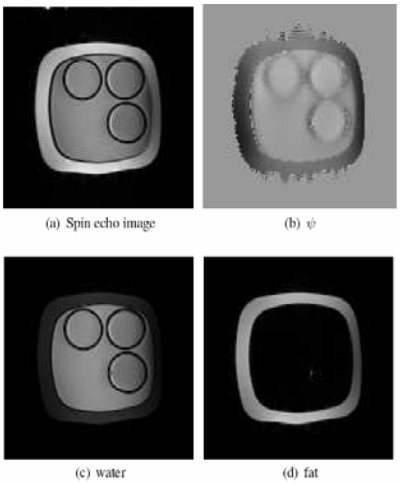

Fig. 1. Fat-water decomposition illustrated on a phantom using spin-echo acquisitions. The phase map is displayed between +300 Hz and

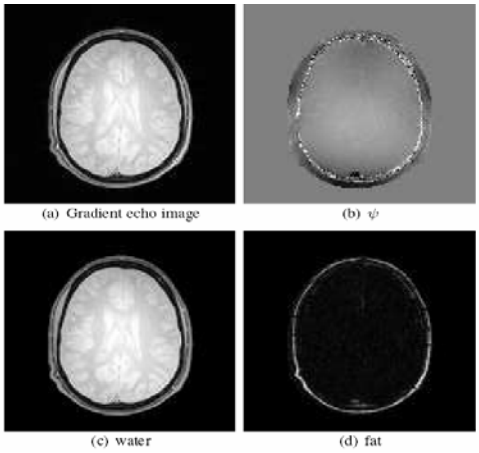

Fig. 2. Fat-water decomposition illustrated on a brain using gradient echo acquisitions. The phase map is displayed between +100 Hz and -100 Hz. The fat image is scaled by 3 as compared to the water image. The study was approved by the Committee of Human Research at UIUC and written informed consent was obtained from the volunteer before the study began.

CONCLUSIONS

We presented a non-iterative algorithm for the decomposition of fat and water. In contrast to classical schemes, our approach is flexible enough to accommodate multiple metabolites and arbitrary sampling patterns. Unlike iterative schemes that converge to a single solution, the proposed algorithm provides a solution set at pixels with multiple solutions; we choose the correct root from this set using the smoothness prior. This approach makes the proposed scheme resilient to local minima and ambiguous estimates.

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