Variational Framework for the Separation of Partially Volumed Tensor Compartments in the Human Brain

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Introduction - White matter delineation and fiber tracking based on DTI are inaccurate for partially volumed voxels [1]. This is since DTI assumes single compartment per voxel. The bi-tensor model can overcome inaccuracies of the DTI model in partially volumed voxels [2]. Fitting the bi-tensor model should allow separation of fiber compartment from mixture of two compartments in partially volumed voxels. However, the inverse problem of bi-tensor model fitting is ill-posed [3]. Here a variational framework is proposed in order to stabilize the fitting procedure of the bi-tensor model. The Multiple Diffusion Tensor Variational (MDTV) framework, previously defined for the mixture of any *n* tensor compartments [4], is applied on nonlinear bi-tensor model fitting. The variational framework allows addition of physiologically driven constraints to pursuit piece-wise smooth fiber orientations. The gradient descent scheme for the framework defines Partial differential equations (PDEs) flow towards best approximation of the solution. This flow is explicitly controlled in order to account for two types of partially volumed voxels: The two fiber population case (such as crossing, kissing or branching fibers), and the case of mixture between fiber compartment and large eigenvalued isotropic compartment (such as CSF contamination or edema). Presented are the MDTV separation results for the two cases, applied on brain data.

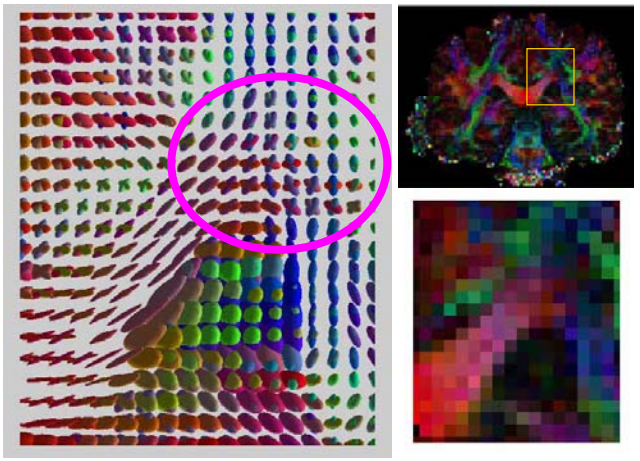

Figure 1: Crossing Fibers. MDTV resolves the intersection between the corpus callosom and cortico-spinal/pontine tracts (magenta ellipse). Each tensor is shown as an ellipsoid colored by the principal orientation (left). The same area in DTI color-coded image (bottom-right) shows a dark hole due to low FA. The ROI is shown as a yellow rectangle on the coronal slice (top-right).

Methodology - The bi-tensor model [2] assumes that the signal decay in DW-MRI is caused by water molecules diffusion within two compartments. The diffusion attenuation for each compartment is assumed to be fully described by a single tensor. The MDTV framework [4] is defined over the bi-tensor model by the following functional:

$$S_{MDTV}(f,\lambda,U) = \alpha S_{MDT} + \int_{\Omega} R(\|\nabla U\|,\|\nabla\lambda\|,\|\nabla f\|)$$

With the data attachment term S_{MDT}, the non-linear least-squares distance of the bi-tensor model from the attenuation signal. The functional is defined over the domain of all possible solutions (Ω). The regularization term operates on the tensor principal eigenvector (U), the vector of three eigenvalues (λ) and the relative volume each tensor compartment occupies (f). The function R in the regularization term is composed by diffusion functions of the form $\phi(s) = \sqrt{1+s^2/K^2}$, for each regularized parameter, and a varying scalar constant K. The effect of the regularization is determined by the scalar α. This type of diffusion functions belongs to the family of "anisotrpic" functions, i.e., they promote piece-wise smooth continuous regions with preserved important edges [5]. Regularization of the principal eigenvector is assumed to pursuit the anatomically expected smooth fiber orientation variation over neighboring voxel. Minimizing the MDTV functional is based on a gradient descent scheme which maintains the tensor's orthonormal basis [4]. The scheme operates on the tensors' spectral decomposition and is explicitly monitored for incorporating additional constraints: In the fiber ambiguity case, each tensor's eigenvalue ratio is restricted not to decrease below a chosen threshold. This ensures that each compartment will be fitted with a cigar shaped tensor possessing FA values in the expected range for fibers. For the CSF contamination one of the compartments is restricted to be isotropic, having tensors with three equal eigenvalues in the range of free water diffusion-coefficient. The remaining tensor compartment is then used for white matter delineation and fiber tracking.

Results - Fiber ambiguity: The healthy volunteer data set presented (Figure 1) was acquired on a 3T (GE) MRI system, using 33 applied gradient orientations, b-value of 1000, 128 X 128 matrix of 1.7mm isotropic voxel dimensions, 72 slices, 4 repetitions and cardiac gating with effective TR of 30 R-R intervals. The image presents part of a coronal slice with an intersection between the corpus callosum (CC) and the cortico-spinal/ cortico-pontine tracts (CST) bundle. In the DTI FA attenuated color coded image the intersection can be seen as a darker area (due to low FA) between the red colored CC and the blue colored CST. This gap is filled in the MDTV resulted ellipsoid visualization (each tensor is an ellipsoid colored by its principal orientation and scaled by its relative volume). Two populations of continuous fibers can be noticed in the expected intersection area. The image shows clear contrast between CSF and brain areas due to the large shapes of ellipsoids in the CSF. However, the image does not show a clear contrast between areas of resolved fiber ambiguity to grey matter. **Edema:** The data set presented (Figure 2) is of a patient suffering from brain tumor surrounded by massive edema. The dataset was a acquired on a 1.5T (GE) MRI system using a conventional 6 gradient orientations scheme, with b-value 1000, TR 10s, 128 X 128 matrix with FOV of 24cm and 48 slices with 3mm slice thickness. The DTI resulted tract shown does not cross the edematous section (shown as a yellowish volume), resulting with poor delineation of CC. The MDTV resulted tract uses the same seed ROI (in the genu of the CC) for fiber tracking. Due to the separation from the free water compartment the tracts pass through the edema and resemble the expected shape of CC.

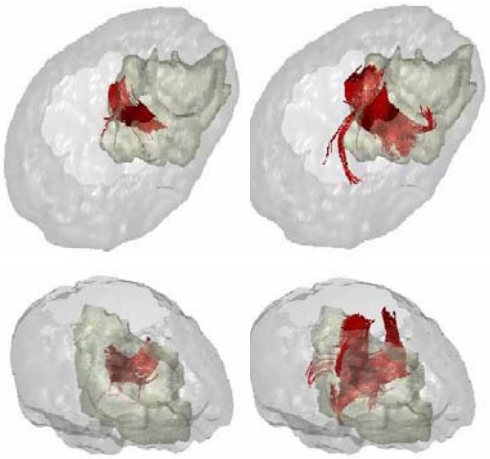

Figure 2: Edema. Fiber tracking through edema using DTI (left) can not pass through the edema due to low FA. The MDTV resulted tracts (right), using the tensor compartment separated from the free water compartment, delineate more of the corpus-callosum.

Discussion - The MDTV framework enables separation of tensor fiber compartments in cases of fiber ambiguity (of two fibers) and contamination with free water compartment. The same framework is used for both cases by altering explicit eigenvalue monitoring. We expect best results to be acquired providing pre-segmentation of white matter from grey matter and pre-segmentaion of partially volumed white matter voxels from homogenous voxels. We also point that the variational framework can be adopted for any other model based diffusion method, and is expected to produce regularized results for the ill-posed inverse problem of fitting such models.

References - [1] Pierpaoli and Basser, MRM 36, 1996. [2] Pierpaoli and Jones, ISMRM, 2004; 12:1215. [3] Alexander *et al.*, MRM 48, 2002. [4] Pasternak *et al.*, In "Visualization and Image Processing of Tensor Fields", Springer, Berlin, 2006. [5] Charbonnier *et al.*, Proc. IEEE ICIP-94,1994.