Quantification of restricted diffusion via kurtosis and q-space imaging

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Introduction

Today it is realized that traditional diffusion imaging methods (based on the ADC and a diffusion tensor in case of anisotropy) do not provide a complete description of the underlying diffusion process. Indeed, for restricted diffusion the probability density function (PDF) is no longer Gaussian as was implicitly assumed in the traditional approach. Therefore, new parameters can be introduced to study the deviation from a Gaussian distribution. Kurtosis has been proposed as a suitable parameter to this end (1). However, kurtosis can be calculated directly from the MR-signal versus b-factor attenuation curves , or via a more general approach on the basis of the PDF's obtained in q-space imaging. In this work we have performed a q-space study on human volunteers and vegetables (asparagus) in order to compare both methods.

Methods

In the short-gradient-pulse approximation, the MR signal is the Fourier transform of the averaged propagator (2):

$$S(q) = \int P(Z,\Delta)\exp(i2\pi qZ)dZ \qquad [1]$$

Here, the q-space coordinate is given by $q = \gamma g \delta$ with g the amplitude of the diffusion encoding gradients, δ the duration of one lobe and Δ the time separation between the onsets of the two lobes in the spin echo sequence. The averaged propagator (or PDF) represents the probability P the spins have diffused a distance Z in a time Δ. The kurtosis of the PDF is given by:

$$k = \frac{\mu_4}{(\mu_2)^2} - 3 \qquad \text{with} \qquad \mu_n = \int Z^n P(Z)\, dZ \qquad [2]$$

because $\mu_1 = 0$ in our case. An alternative way to calculate the kurtosis is via the MR signal attenuation in terms of the b-factor (1):

$$\ln[S(b)] = \ln[S(0)] - bD + \frac{1}{6}b^2D^2K \qquad [3]$$

wherefrom the apparent diffusion coefficient D and apparent kurtosis K can be obtained using regression. Equation[3] corresponds to making a fourth order cumulant expansion of the MR signal or, equivalently, making a fourth order Gram-Charlier expansion of the PDF (3).

Diffusion measurements were performed on a clinical Philips Achieva 3T scanner with maximum gradient strength 60 mT/m. The SE-EPI sequence had diffusion weighting for 16 b-values in the range 0 – 22 000 s/mm^2 (or equivalently: 16 equisdistant q-values in the range 0 – 110 mm^{-1}) and for 6 diffusion directions. Five slices (FOV=230mm, 112^2 -matrix and 5 mm slice thickness) were measured on volunteers in 9 min. On aparagus, more b-values (100 up to 25 000 s/mm^2) and a better spatial resolution were used (the loss of SNR was compensated by signal averaging). Measurements were performed for different diffusion timings $\varepsilon = \delta/\Delta$ and voxel sizes. A q-space analysis yielded the PDF's wherefrom Return-To-Origin (RTO) $P(Z=0)$, Full-Width-at-Half-Maximum (FWHM) and kurtosis maps were constructed. The estimation of K via eq.[3] was done on the basis of 6 b-values in the range 0-3500 s/mm^2 (1).

Results and conclusions:

The figure shows one slice (one direction) from a volunteer ($\varepsilon = 31.3/51.0$). Shown are the b=0 image and maps for RTO-probability, FWHM w in μm , kurtosis k, ADC D in $\mu m^2/s$ and apparent kurtosis K. It is clear that both methods do not result in identical kurtosis-values. More contrast can be observed in maps reconstructed on the basis of the PDF. Moreover, the values obtained using eq.[3] seem to be inconsistent: for instance in the ventricles, a kurtosis close to zero is expected (which is obtained in the k-map but not in K-map). The fact that incorrect results are obtained from eq.[3] could be expected because a truncated cumulant expansion can not well model all situations. Therefore, we prefer to estimate the parameters on the basis of a more general q-space analysis. Both methods contain systematic errors due to gradient timings(4). However, it was found that q-space analysis always leaded to superior results for all studied $\varepsilon = \delta/\Delta$ -values.

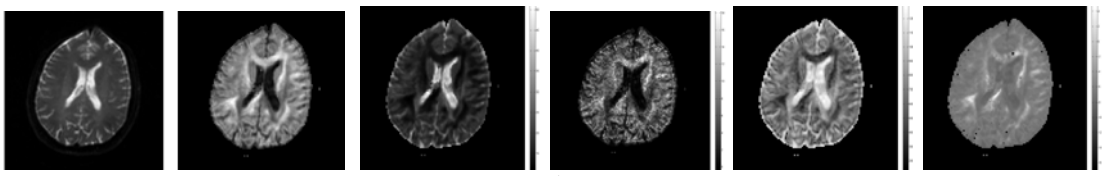

Figure: b=0 image and maps for RTO, w in μm , k, D in $\mu m^2/s$ and K.

References:
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