Entropy-based Characterization of Diffusion Anisotropy

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Introduction The most commonly used approach in modeling water diffusion in living tissue is through a single tensor in diffusion tensor imaging (DTI) (1). The main shortcoming is that the diffusion profile is not always Gaussian as DTI assumes especially in voxels with partial voluming or having multiple fiber orientations (2). The single tensor model is inadequate in such cases leading to inaccurately calculated anisotropy indices like fractional anisotropy (FA) (3). Alternative approaches include diffusion spectrum imaging (DSI) (4) and Q-Ball imaging (QBI) (5). In this paper, we describe an approach based on the information content inherent in the diffusion attenuation values measured across the applied diffusion-sensitizing gradient directions. We calculate the information theoretic measure, Shannon's entropy (6), on the attenuation values to directly characterize diffusion anisotropy from the diffusion-weighted magnetic resonance images.

Methods Shannon's entropy represents the average amount of information per message relayed through a communications channel as described by $H(X) = -\sum p(x_i)\log p(x_i)$, -where X is a random variable, $p(x_i)$ is the probability of outcome x_i, and $H(X)$ is the entropy of random variable X. For a given number of outcomes n, the maximum entropy is equal to $\log(n)$, which occurs when all probabilities are equal and is intuitively the most uncertain situation. If all the $p(x_i)$ but *one* are zero, the non-zero one has a probability of 1, then the corresponding entropy is equal to *zero.* Thus only when we are certain of the outcome does entropy vanish. Otherwise entropy is positive (6). The outcomes x_i are taken here as the attenuation values in each voxel resulting from the diffusion gradient directions and entropy is calculated on a voxel-by-voxel basis.

Attenuation values in isotropic tissue such as cerebrospinal fluid (CSF) will be independent of the diffusion gradient direction. Therefore a histogram of attenuation values for a CSF voxel will be sharply peaked, indicating that these values have a high probability of occurrence across all the diffusion directions and have an entropy value close to zero. This, however, is not the case with tissue that is slightly anisotropic like brain gray matter (GM) or tissue that is highly anisotropic such as brain white matter (WM). For a highly structured tissue like WM, the attenuation values will strongly depend on the direction along which the diffusion gradient is applied. Histograms for WM attenuation values are broader, indicating that information is gained when the gradient directions are varied in space (7).

A healthy normal male volunteer was scanned using a Siemens 3.0 Tesla Trio employing a diffusion-weighted single-shot EPI sequence (courtesy Massachusetts General Hospital). Diffusion-weighted images were acquired along 60 directions with two b-values ($b \approx 0$ and 1000 s/mm^2) -with imaging parameters: TR = 8000 ms, TE = 104 ms, field of view (FOV) = 256 mm x 256 mm, matrix size of 128 x 128, resolution of 2 x 2 x 2 mm^3, and phase partial Fourier of 6/8. Three consecutive acquisitions were made for subsequent averaging. Skull stripping, eddy current correction and DTI analysis were performed using FSL software (fMRIB, Oxford, UK, www.fmrib.ox.ac.uk/fsl/). All entropy analysis was performed through in-house software written in MATLAB (Mathworks, Inc., Natick, Massachusetts).

Results Tissues with different diffusion anisotropy display differing ranges of entropy as shown in Fig. 1(b). The difference in entropy values between various tissue types (CSF, GM, thalamus, caudate nucleus, and WM) is a direct result of their different attenuation value histograms, Fig 2. A plot of mean FA values in the outlined ROIs in Fig. 1(c) versus the mean entropy is shown in Fig. 3, revealing an expansion of the differences between CSF and gray matter.

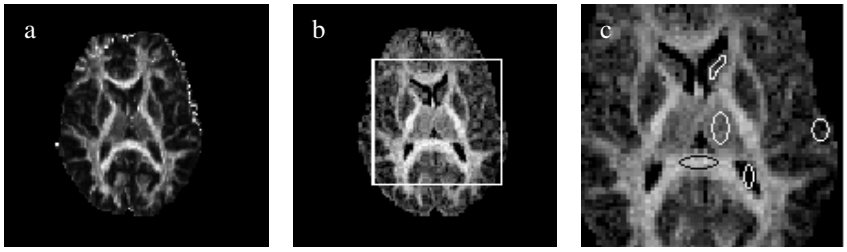

Fig. 1 (**a**) Fractional anisotropy (FA) map and (**b**) entropy map. Area in white box enlarged in (**c**) WM ROI marked in black, CSF, thalamus, caudate nucleus and GM ROIs in white.

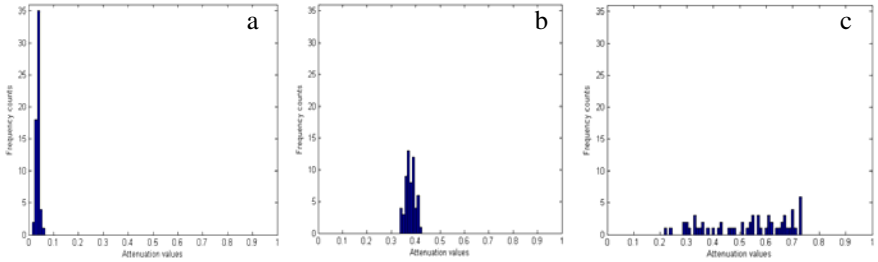

Fig. 2 Representative voxel attenuation values histogram of CSF (**a**), GM (**b**), and WM (**c**).

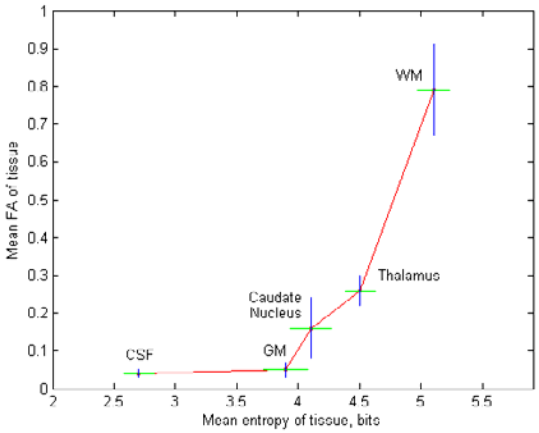

Fig. 3 Plot of mean entropy vs. mean FA for various tissue types. The blue vertical lines represent +/− one standard deviation of FA and the green horizontal lines represent +/− one standard deviation of entropy.

Discussion We have introduced a new measure of diffusion anisotropy based on information content. Our method is model-free and assumes no *a priori* knowledge of the underlying diffusion process making it less biased. As we can see from Fig. 3, the measure provides additional contrast between different tissue types by stretching the range between CSF and GM which is not evident with FA while slightly compressing in other ranges. This approach however needs a relatively high number of uniformly spaced samples.

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References

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