Roughness: A reshuffling-variant differential geometric index for DWI

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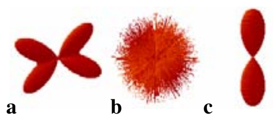
INTRODUCTION

In the past few years, there has been great interest in developing methods that are more general than diffusion tensor imaging (DTI) to model the orientational dependence of the diffusion MR signal. Most techniques that have been proposed require data that is sampled on the surface of a unit sphere and/or the data is transformed into a directional profile of a certain quantity. There has also been some interest in reformulating the scalar measures (such as anisotropy) derived from the diffusion tensor to better quantify the information on orientational preference present in the MR signal. For example, the generalized anisotropy (GA) measure [1] has been proposed that depends solely on the variance of the orientation dependent function. Since indices like fractional anisotropy (FA) and relative anisotropy (RA) are computed by normalizing the variance of the diffusion tensor's three eigenvalues, GA can be thought of as a generalization of FA and RA.

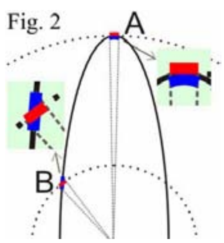


a b c

Fig 1. **(a)** An orientation profile depicting two crossing fibers. **(b)** The values used in generating the profile in (a) redistributed on the sphere randomly. **(c)** The redistribution was done so that higher values are assigned to the z-direction. In all three cases, the profiles have the same variance value.

In this work, we introduce the "roughness" measure as a new index that generalizes a different class of anisotropy indices. An example for an index that belongs to this class in the context of DTI is: $A_{\text{major}} = \dfrac{\lambda_1 - (\lambda_2 + \lambda_3)/2}{\lambda_1 + \lambda_2 + \lambda_3}$ introduced in ref. [2]. The defining feature of this class of indices is that they are "reshuffling variant." For more indices that belong to this class see ref. [3]. In the case of A_{major}, if one interchanges the value of λ_1 with that of λ_2, then A_{major} changes. However, indices like FA and GA that depend only on the dispersion of the (eigen)values can be considered *global* indices, hence they are reshuffling invariant. Fig. 1 shows three different distributions of the same profile on the unit hemisphere. Although they look quite different, they all yield the same variance value.

THEORY



Let $\mathbf{X}(\theta,\varphi)=r(\theta,\varphi)\,(\sin\theta\cos\varphi\,,\ \sin\theta\sin\varphi\,,\ \cos\theta)^T$ be the parametrized surface that one is computing to visualize the anisotropy of the data where $r(\theta,\varphi)$ is the distance from the origin along the direction specified by the spherical coordinates θ and φ. Note that $r(\theta,\varphi)$ can be taken to be the signal or ADC values from a HARDI experiment. Alternatively, they may be taken, for instance, to be the orientation distribution function (ODF) obtained via q-ball imaging (QBI) and diffusion spectrum imaging (DSI); or the probability profiles of diffusion orientation transform (DOT). Then the roughness measure is defined to be

$$R = \frac{\text{surface area}}{\text{spherical area}} = \frac{\int \sqrt{g}\ \mathrm{d}\theta\,\mathrm{d}\varphi}{\int r(\theta,\varphi)^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi},$$

where g is the determinant of the metric tensor with components $g_{ij} = \dfrac{\partial \mathbf{X}}{\partial u_i} \cdot \dfrac{\partial \mathbf{X}}{\partial u_j}$ where $u_1 = \theta$ and $u_2 = \varphi$.

Note that the roughness index has been used to quantify the complexity of molecular surface shapes in the computational chemistry literature [4].

Figure 2 shows an analogous two-dimensional geometry. Here, a portion of the surface whose roughness is to be estimated is depicted with the solid line. The thick red lines show the unit area (arclength in 2-d space) elements of the spherical area at the points A and B, where the blue lines show the same for the true surface area. Note that point A contributes almost equally to the two definitions of area. However, at point B the blue line is longer than the red line. For a sphere, the two areas are equal; this results in a roughness value of 1. As the surface gets elongated (even locally) R will get larger and may increase without bound. Therefore, it would be convenient if the roughness index is mapped onto the interval [0,1]. We achieve this through the transformation $NR=(\tanh[4(R-1)])^2$, where NR stands for "normalized roughness."

RESULTS

All results shown are obtained from a data set acquired with 2.5 x 2.5 x 2.5 mm^3 isotropic resolution using a 1.5-T GE (General Electric, Milwaukee, WI) MRI scanner. A diffusion-weighted EPI pulse sequence was used. After a single acquisition is performed with no diffusion-weighting, 50 images with diffusion-weighting gradients oriented along different directions were acquired at b=1100 s/mm^2.

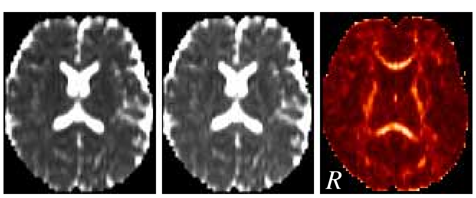


Fig. 3 From left to right: spherical area, surface area and roughness computed on a brain slice. The orientation dependent function used was the 6-th order spherical harmonic representation of the ADC profile obtained from an HARDI-type acquisition. The contrast in the spherical area image is similar to the mean diffusivity (MD) index commonly used to depict an orientation independent diffusivity map. However, the spherical area map is obtained by integrating the square of the ADCs rather than the ADC profile itself as is done in the computation of MD. Although the contrast is similar in the surface area map, in voxels with rougher ADC profiles, the values are larger than those in the spherical area map. The roughness map is just the ratio of the two area maps and highlights the regions that have global or local elongated shapes.

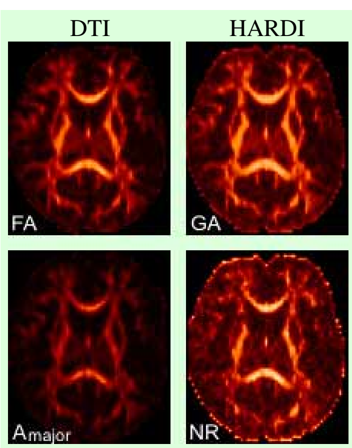


Fig. 4 Four indices that were discussed in this work. The first row includes the indices FA and GA that are computed from the variances of the eigenvalues of the diffusion tensor and ADC profiles respectively. The second row shows the indices that are reshuffling-variant. In the case of A_{major}, the value of the index is not constant under the interchange of two eigenvalues. Similarly, NR value changes if one interchanges the ADC values along two different orientations. It is clear that the NR index shows more details compared to the other indices indicating its sensitivity to finer details due to local variations of the ADC values on the surface of the sphere.

DISCUSSION

In this work, we presented a new measure that quantifies the orientational complexity of multidirectional diffusion-weighted MR data. The proposed measure can be used with many different techniques such as, DTI, QBI, DOT, DSI and spherical deconvolution. The important feature of the index is that its value depends on the derivatives hence the local characteristics of the surface. To our knowledge, the only other differential geometric measure that has been used in diffusion-weighted imaging is the geodesic anisotropy index [5] that employs the geometry of the space of diffusion tensors. However, the roughness index is different in that, it uses the geometry of the surface whose complexity is to be quantified and as such opens a new window in the characterization of the features of the diffusion-weighted signal profiles.

References:
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