Tensor Estimation for DTI Using Non-Linear Conjugate Gradient

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Introduction Reconstruction of diffusion tensors from multi-shot & multi-coil diffusion tensor imaging data is generally a two step procedure : 1) reconstruction of individual diffusion weighted images using non-linear phase correction and SENSE reconstruction [1],[2] ; 2) estimation of tensors from the diffusion weighted images using multivariate regression. However, in the case of rotational motion between the shots, each interleaf experiences a different diffusion encoding. This makes it impossible to reconstruct each diffusion weighted image individually. In this study, we propose a single-step tensor estimation scheme that uses a non-linear conjugate gradient (NLCG) algorithm to overcome this problem.

Materials and Methods For a multi-shot & multi-coil DTI acquisition, the k-space signal is given by:

$$\underbrace{d_{\gamma,\delta}\left(\mathbf{k}_{\kappa,\xi}\right)}_{\substack{\textbf{Acquired}\\ \textbf{k-space data}}} = \frac{1}{n_\rho}\sum_\rho \underbrace{e^{-j\left(\mathbf{k}_{\kappa,\xi}\right)^T\mathbf{r}_\rho}}_{\substack{\textbf{Gradient}\\ \textbf{encoding}}} \underbrace{s_{\gamma,\xi}\left(\mathbf{r}_\rho\right)}_{\substack{\textbf{Sensitivity}\\ \textbf{encoding}}} \underbrace{e^{-\sum_{i,j}\left[\mathbf{b}_{\delta,\xi}\right]_{i,j}\left[\mathbf{D}\left(\mathbf{r}_\rho\right)\right]_{i,j}}}_{\substack{\textbf{Diffusion}\\ \textbf{encoding}}} \underbrace{m\left(\mathbf{r}_\rho\right)}_{\substack{\textbf{b=0}\\ \textbf{image}}}$$

where $\boldsymbol{\gamma}$ stands for the coil number, $\boldsymbol{\delta}$ for diffusion weighting direction number, $\boldsymbol{\xi}$ for interleaf number, $\boldsymbol{\kappa}$ for k-space point and $\boldsymbol{\rho}$ for image space point. $m(\mathbf{r}_\rho)$ is the non-diffusion weighted image that can be obtained with the conventional SENSE reconstruction and is assumed to be known. Because of the random nonlinear phase and altered sensitivity exposure due to motion, each interleaf is assigned a different set of modified coil sensitivity profiles [3]. For such a situation, the diffusion encoding ($\mathbf{b}_{\delta,\xi}$) has to be different for each interleaf as well. In particular, when there is rotational motion, $\mathbf{b}_{\delta,\xi}$ is given by $\mathbf{b}_{\delta,\xi} = \mathbf{R}_{\delta,\xi}\,\mathbf{b}_\delta\,\mathbf{R}_{\delta,\xi}^{\ T}$, where $\mathbf{R}_{\delta,\xi}$ is the rotation matrix of $\boldsymbol{\xi}^{th}$ interleaf with respect to a template. In the equation above, we assume that the k-space data $\mathbf{d}_{\gamma,\delta}$ has already been corrected for translational motion by applying a linear phase. The k-space trajectories $\mathbf{k}_{\kappa,\xi}$ are also assumed to have been counter-rotated accordingly to account for rotation.

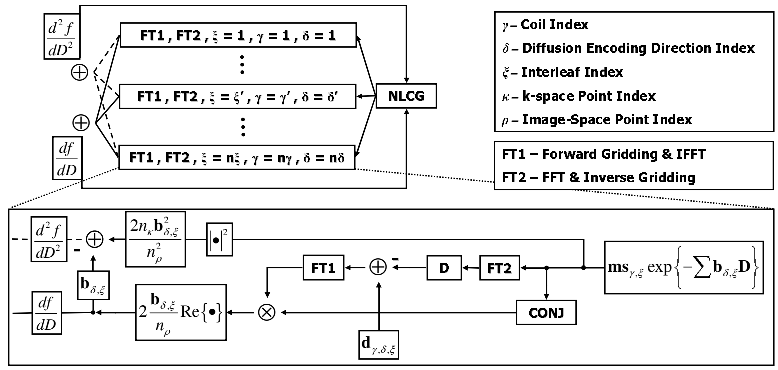

Figure 1 – *The Nonlinear Conjugate Gradient Algorithm for Diffusion Tensor Estimation. The main algorithm is shown with the flow diagram on the top left corner. Each channel corresponds to a specific interleaf, coil and diffusion encoding direction. An enlarged version of the individual blocks is shown at the bottom.*

The aim of the NLCG algorithm is to find the $\mathbf{D}(\mathbf{r}_\rho)$ that minimizes a cost function *f*. For this purpose, we define the cost function as the sum-of-squares of the differences between the measured and calculated k-space values over all coils, diffusion encoding directions, interleaves and k-space points:

$$f\left(\mathbf{D}\left(\mathbf{r}_\rho\right)\right) = \sum_{\gamma,\delta,\xi,\kappa}\left| d_{\gamma,\delta,\xi}\left(\mathbf{k}_{\kappa,\xi}\right) - \frac{1}{n_\rho}\sum_\rho e^{-j\left(\mathbf{k}_{\kappa,\xi}\right)^T\mathbf{r}_\rho} s_{\gamma,\xi}\left(\mathbf{r}_\rho\right) e^{-\sum_{i,j}\left[\mathbf{b}_{\delta,\xi}\right]_{i,j}\left[\mathbf{D}\left(\mathbf{r}_\rho\right)\right]_{i,j}} m\left(\mathbf{r}_\rho\right)\right|^2$$

To find the $[\mathbf{D}(\mathbf{r}_\rho)]_{i,j}$ that minimizes this cost function, we employed the NLCG algorithm using Polak-Ribiere and Newton-Raphson line search [4]. In order to use this algorithm efficiently, we need the first derivative and an approximation to the second derivative of the cost function given above with respect to the tensor elements $[\mathbf{D}(\mathbf{r}_\rho)]_{i,j}$. It is required that these first and second derivatives are obtained for all image points and tensor elements. This requires the summation above to be performed $\boldsymbol{6} \times \boldsymbol{n_\rho}$ times. This calculation can be carried out more efficiently by approximating the summation over $\boldsymbol{\kappa}$ using inverse gridding and FFT. With this technique, the first and second derivatives of the cost function with respect to the element $(\boldsymbol{i_1,j_1})$ of the diffusion tensor at location $\boldsymbol{r_{\rho 1}}$ becomes :

$$\frac{df\left(\mathbf{D}\left(\mathbf{r}_\rho\right)\right)}{d\mathbf{D}_{i_1,j_1}\left(\mathbf{r}_{\rho_1}\right)} \approx \frac{1}{n_\rho}\sum_{\gamma,\delta,\xi} 2\left[\mathbf{b}_{\delta,\xi}\right]_{i,j}\operatorname{Re}\left\{\mathbf{DWI}^*_{\gamma,\delta,\xi}\left(\mathbf{r}_{\rho_1}\right)FT1\left\{d_{\gamma,\delta,\xi}\left(\mathbf{k}_{\kappa,\xi}\right) - FT2\left\{\mathbf{DWI}_{\gamma,\delta,\xi}\left(\mathbf{r}_{\rho_1}\right)\right\}\right\}\right\}$$

$$\frac{d^2f\left(\mathbf{D}\left(\mathbf{r}_\rho\right)\right)}{d\mathbf{D}_{i_1,j_1}\left(\mathbf{r}_{\rho_1}\right)^2} \approx \sum_{\gamma,\delta,\xi}\left\{\left[\frac{2n_\kappa\left[\mathbf{b}_{\delta,\xi}\right]^2_{i_1,j_1}}{n_\rho^2}\left|\mathbf{DWI}_{\gamma,\delta,\xi}\left(\mathbf{r}_{\rho_1}\right)\right|^2 - \left[\mathbf{b}_{\delta,\xi}\right]_{i_1,j_1}\frac{df\left(\mathbf{D}\left(\mathbf{r}_\rho\right)\right)}{d\mathbf{D}_{i_1,j_1}\left(\mathbf{r}_{\rho_1}\right)}\right]\right\}$$

where $\mathbf{DWI}_{\gamma,\delta,\xi}\left(\mathbf{r}_\rho\right) = s_{\gamma,\xi}\left(\mathbf{r}_\rho\right) e^{-\sum_{i,j}\left[\mathbf{b}_{\delta,\xi}\right]_{i,j}\left[\mathbf{D}\left(\mathbf{r}_\rho\right)\right]_{i,j}} m\left(\mathbf{r}_\rho\right)$

A flowchart of our algorithm is given in Figure 1. Following the convention in [1], the FFT & Inverse Gridding and Forward Gridding & IFFT operations are denoted by FT2 and FT1 respectively. In order to test our method, *in-vivo* experiments were carried out with TR/TE=3000/61 ms, 6 diffusion gradient directions [$(1\ 1\ 0)^T$,$(1\ 0\ 1)^T$, $(0\ 1\ {-1})^T$, $({-1}\ 1\ 0)^T$, $(0\ 1\ 1)^T$, $(1\ 0\ {-1})^T$], NEX=2, matrix size = 128x128 and 8 interleaves. A spiral-in navigator was used to measure the amount of motion for each interleaf. The performance of NLCG algorithm was evaluated for the cases with and without subject motion.

Results and Discussion Figure 2 shows the FA maps obtained by NLCG. It can be seen that the visualization of white matter pathways is successful. The moderate noise in the images reconstructed with NLCG is a result of the sensitivity of CG methods to noisy data. This can be corrected with appropriate preconditioning ([1],[4]) but was not explored in this work. The rigid head motion related artifacts are significantly removed by the proposed motion correction scheme.

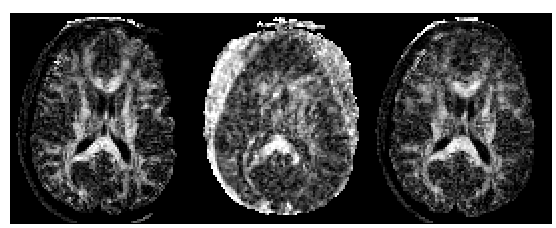

Figure 2 – *In-vivo results showing the FA maps reconstructed with NLCG. The FA map reconstructed using NLCG in case of no subject motion (left) demonstrates the successful visualization of white matter pathways. In the presence of motion, the FA map reconstructed with SENSE and without motion correction (middle) shows significant motion artifacts compared to the motion-corrected image using NLCG (right).*

Conclusion A one-step method for combined motion correction and tensor estimation using Non-linear Conjugate Gradient is proposed. The current method performs similar to the conventional two step tensor estimation when there is no motion. The consecutive FFT, inverse gridding, forward gridding and IFFT operations applied here are similar to the algorithm used in generalized SENSE reconstruction for arbitrary trajectories [4]. Thus, the reconstruction time per image is mainly determined by the forward and inverse gridding steps and is comparable to that of non-linear phase correction given in [2]. Further speed-up is possible by combining the forward and inverse gridding operations and using the transfer function approach [5].

References [1] Pruessman et al, MRM, 46:638-51,2001 [2] Liu et al, MRM, 54:1412:22, 2005 [3] Bammer et al, MRM, in press [4] Shewchuk, “An Introduction to the Conjugate Gradient Method Without the Agonizing Pain”, 1994 [5] Wajer et al, ISMRM 2001, 767
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