Optimum *b*-Value vs. SNR for Apparent Diffusion Coefficient Measurements

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Introduction: Apparent diffusion coefficient (*ADC*) measurements generally consist of two-point estimation schemes, where diffusion-weighted (DW) images are acquired at two different *b*-values, b_1 and b_2. The optimization of *b*-value selection has previously been investigated [1-4], assuming high enough signal-to-noise ratio (*SNR*), for which the noise is Gaussian so that the error propagation (EP) formulas are valid. However, for low *SNR*, the EP method is not applicable. In this work, utilizing the true statistics of noise, we calculate the optimum *b*-value as a function of SNR. The results presented in this work become especially important for high-resolution DW imaging, which intrinsically suffers from low SNR. We apply our method to DW imaging of the spinal cord at high resolution to demonstrate the improvement in the resulting DW images and ADC maps.

Theory: Our goal in this work is to address how the differences between EP estimation and the exact statistics of noise in ADC maps influence the optimum *b*-value selection. Figure 1 shows the calculated true variance and mean for $v = \ln|1 + n/S|$, and the EP estimation of these statistics. Here, *v* is the noise that arises after taking the logarithm of the signal in order to calculate ADC, *S* is the true signal and *n* is the complex-valued noise, with both the real and imaginary components having a zero-mean and σ^2-variance Gaussian distribution. Using the Rician noise statistics for $|1 + n/S|$, the variance and the mean of *v* can be shown to depend only on $S/\sigma=SNR$. To emphasize this, we denote the variance and mean of *v* as $\sigma_v^2(SNR)$ and $\mu_v(SNR)$. Figure 1 shows that the calculated true statistics are in excellent agreement with phantom experiment results and that the EP method is inaccurate for low SNR.

The optimization of *b*-value selection is done by optimizing SNR of the ADC (denoted as *DNR*) with respect to $D\Delta b$, where *D* is the true diffusion coefficient and $\Delta b = b_2 - b_1$. Without loss of generality, we can assume $b_1<b_2$ (in practice, $b_1\approx 0$). Furthermore, the ratio of the number of images acquired at b_2 and b_1 ($r_{21}=N_2/N_1$) has to be optimized as well. The EP method gives the optimum $D\Delta b$ as 1.28, with optimum $r_{21} = e^{D\Delta b} \approx 3.59$ [1]. The fact that the EP method fails to estimate the statistics of *v* for low *SNR* implies that the optimum $D\Delta b$ is not a constant, but a function of *SNR*.

For a general solution, the optimum $D\Delta b$ should satisfy $\partial DNR/\partial(D\Delta b) = 0$. To simplify the optimization, we define $SNR_{total} = SNR_1\sqrt{N_{total}}$, where SNR_1 is the SNR of a single image at b_1 and $N_{total}=N_1+N_2$. Using the true $\sigma_v^2(SNR)$ for this optimization shows that for $SNR_{total}<12.85$, there is no solution that satisfies $\partial DNR/\partial(D\Delta b) = 0$. This is due to the fact that $\sigma_v^2(SNR)$ converges to a constant value (~0.41) as *SNR* goes to zero, which results in *DNR* increasing linearly with $D\Delta b$. However, at these high $D\Delta b$ and low SNR points, $\mu_v(SNR)$ grows very rapidly, introducing a large error in the estimation of *ADC*. To avoid this error, there needs to be a criterion to limit the maximum allowable $D\Delta b$, or equivalently to set a lower threshold for the SNR of the DW image with the higher b-value (i.e., $SNR_{b2} = SNR_2\sqrt{N_2}$). Since the minimum SNR_{total} that has a solution for $\partial DNR/\partial(D\Delta b) = 0$ is 12.85, we can use SNR_{b2} at that point (which is equal to 3.43) as a lower threshold. Setting this as the lower threshold for SNR_{b2} results in the optimum $D\Delta b$ vs. SNR_{total} curve shown in Figure 2.

As expected, the proposed optimum $D\Delta b$ curve approaches the asymptotic EP solution $D\Delta b\approx 1.28$ for high SNR. Note that the points above $SNR_{total}=12.85$ have true optimal solutions, and the points below $SNR_{total}=12.85$ use the highest $D\Delta b$ they can afford without falling below the lower threshold of $SNR_{b2}=3.43$. For $SNR_{total}>12.85$, $r_{21}=e^{(D\Delta b)^*}$ is the optimal ratio. For $SNR_{total}<12.85$, the optimum ratio is slightly larger than $e^{(D\Delta b)^*}$. The green dash-dot curve in Figure 2 is the result of setting $r_{21}=e^{(D\Delta b)^*}$ for $SNR_{total}<12.85$, which yields very close to optimum results. This approximate solution has a closed form expression: $D\Delta b = \ln(2/(R + \sqrt{R^2 + 4R}))$ where $R=(3.43/SNR_{total})^2$. Figure 2 also implies that in order to talk about an optimum diffusion measurement, SNR_{total} should be kept above 4.85, the point where the $(D\Delta b)^*$ becomes zero.

Results: Figure 3 summarizes the proposed method for optimum *b*-value selection. To demonstrate the improvement achieved with this method, high-resolution (0.94x0.94 mm^2 in-plane resolution, 9x4.5 cm^2 FOV) single-shot DW imaging of the cervical spinal cord of a healthy volunteer was performed on a 1.5T GE Excite scanner. For $SNR_1=1.5$, a total of 23 images were acquired, i.e., $SNR_{total}\approx 7.5$. From Figure 2, the optimum $D\Delta b$ for this SNR_{total} is $(D\Delta b)^*\approx 0.63$ ($r_{21}^*\approx 1.88$, $N_1=8$, $N_2=15$). These parameters were compared against the EP method $D\Delta b\approx 1.28$ ($r_{21}\approx 3.59$, $N_1=5$, $N_2=18$). Effects of T_2 decay on b-value selection [4] were ignored. Therefore, TE was kept the same (74 ms), in order to avoid T_2 effects on the comparison. The results of high-resolution DW spinal cord imaging are shown in Figure 4. Note that the DW image has improved and the ADC map exhibits noticeably higher SNR.

Discussion: In [5], it was mentioned that the DW signal should always be greater than the mean of the background noise. This corresponds to a criterion $SNR_{b2} > \sqrt{\pi/2}$, which yields the same result as the EP method for $SNR_{total}>5.1$. Therefore, the results provided in Figure 4a and 4c can also be seen as the results of this criterion. Our observation is that the lower threshold criterion for SNR_{b2} needs to be more conservative than $\sqrt{\pi/2}$.

Conclusion: It is shown that the optimum $D\Delta b$ depends on the *SNR* of the imaging scheme, as shown in Figure 2. The results presented in this work become especially important for high-resolution DW imaging, which intrinsically suffers from low SNR.

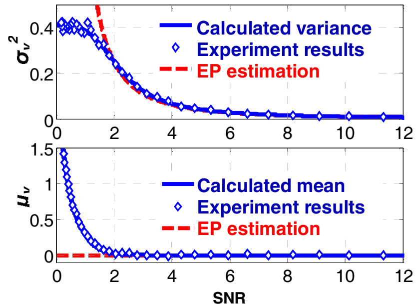

Figure 1. (a) *Variance and* ***(b)*** *mean of* $v = \ln|1 + n/S|$ *as a function of SNR.*

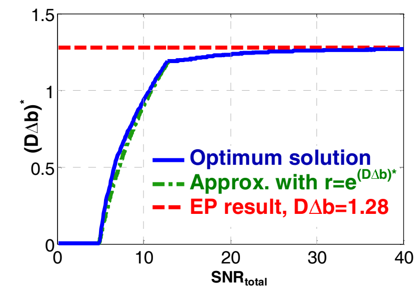

Figure 2. *Optimum* $D\Delta b$ *vs.* SNR_{total} *curve*

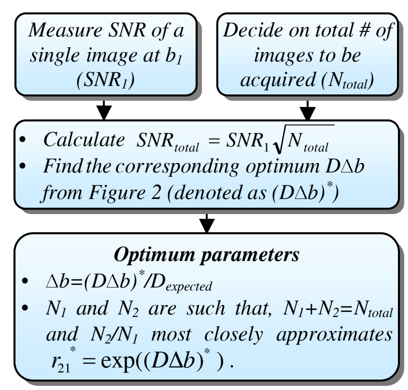

Figure 3. *Flow chart summarizing the optimum b-value selection.*

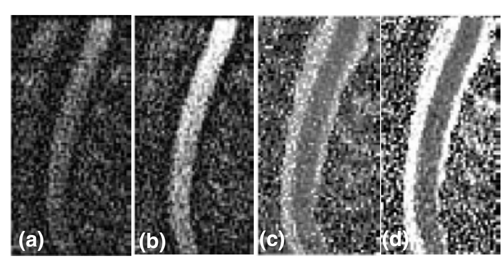

Figure 4. *Comparison at low SNR level.* $SNR_1\approx 1.55$, $N_{total}=23$, *i.e.,* $SNR_{total}\approx 7.5$. *DW images of cervical spinal cord at* ***(a)*** *b=710s/mm*^{2} *(*$D\Delta b\approx 1.28$*) and* ***(b)*** *b=350s/mm*^{2} *(*$D\Delta b^*\approx 0.63$*).* ***(c)*** *and* ***(d)*** *are the corresponding ADC maps. DW image (b) has improved when compared to (a), and ADC map (d) has noticeably higher SNR than (c).*

References:

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