Evaluation of the higher order tensor estimation quality for established gradient encoding schemes

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Introduction:

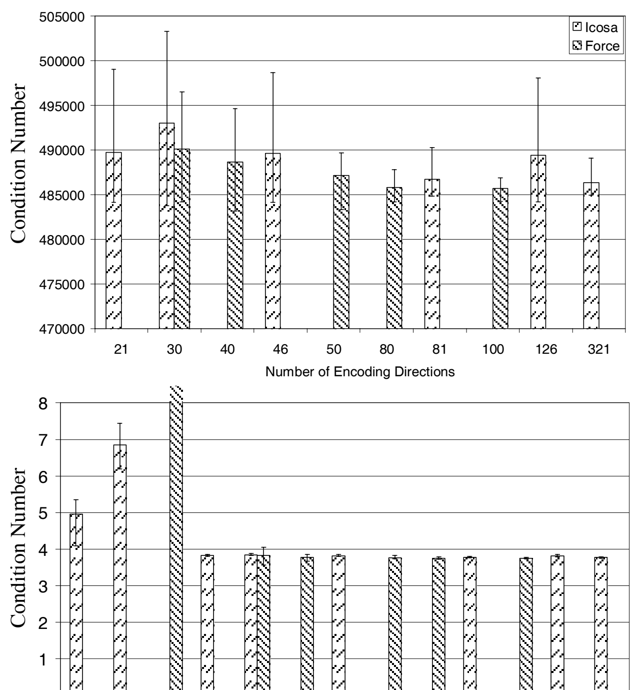
Complicated intra-voxel fiber structures such as crossing or branching fiber bundles are one reason for non-Gaussian diffusion, which cannot be correctly described by second order diffusion tensors (DT), see for example [1]. To overcome this problem higher order diffusion tensor models [2], [3] were proposed, that require HARDI measurements with a large number of encoding directions. The increase in independent tensor elements leads to an increase in the rank of the estimation-matrix and its respective condition number for the tensor evaluation. In this work we present a comparison of established gradient encoding schemes (GES) for the higher order tensor (HOT) estimation. The focus here is on the icosahedral and force-minimizing encoding schemes [4], [7]. We show that not all of them are equally well suited for HOT evaluations.

Methods:

Our analysis examines GES for the evaluation of two HOT models known from literature, the generalized diffusion tensor by Oezarslan and Mareci [3] and the tensor hierarchy introduced by Liu et al. [2]. The first tensor model is limited to reconstructing symmetric fiber constellations and does not allow an immediate reconstruction of their fiber orientations [5]. In contrast to this model the tensor hierarchy is able to derive fiber orientations directly and can represent asymmetric fiber constellations due to the use of odd order tensors, which are solely determined from the imaginary part of the complex measured signal. For both models the tensor can be estimated (for each voxel) by solving an equation system of the form: $\mathbf{D} = \mathbf{B}^{-1}\mathbf{Y}$. Here **D** is a vector comprising all tensor elements, **Y** stores the logarithmic signal ($\log(S/S_0)$) and **B** is the so-called estimation or **B**-matrix, which contains the information on the encoding scheme and the diffusion weighting. In this evaluation of GES we focus on the comparison of the condition numbers of the corresponding **B**-matrices, which give an upper boundary for the error propagation in the tensor estimation [8]. This measure and its rotational variance are commonly used to compare different encoding schemes [6], [8]. The force-minimizing scheme was evaluated 100 times for each number of encoding directions to obtain reliable estimates of the the mean condition number and its standard deviation.

Results:

Both models were evaluated for a (maximal) tensor order of six. In Fig. 1 the results for the evaluation for the maximal tensor order four are exemplarily shown. In our evaluations, we observed, that the HOT estimation stabilizes (in terms of the condition number variance) in general with an increase in encoding directions. This stabilization is particularly salient in the force-minimizing schemes for single HOT [3] estimation, which stabilize abruptly when the number of encoding directions is at least twice the number of independent tensor elements (see Fig. 1 (bottom); here this is 15). Figure 1 (top) illustrates the rotational dependence of the icosahedral GES as error-bars and (connected) points. It can also be seen that for these schemes the condition number does not necessarily decrease with an increase in encoding directions. There is a continuous decrease in the schemes resulting from even order icosahedral triangulations (21, 81, 321). The reason why other icosahedral schemes do not improve in the same way needs to be further investigated. One explanation could be the existence of different sub-families of the icosahedral schemes (for example even- and odd-order triangulations) that stabilize at individual rates. It was also observed that the icosahedral 'bucky-ball' scheme with 30 directions could not estimate the single HOT model [3] for order six, because **B** did not reach the full rank of 28, i.e. a large number of gradient directions alone does not guarantee that the HOT models can be applied. Figure 1 (top) illustrates clearly that the force-minimizing GES are favorable for evaluations of HOT hierarchies [2], because they stabilize considerably faster than the icosahedral schemes. When the number of encoding directions is sufficiently high the force-minimizing schemes will outperform the icosahedral ones for the single HOT model [3] as well.



	15	16	20	21	30	40	46	50	80	81	100	126	321
Icosa	4.9509	6.8550		3.8172	3.8376		3.8151			3.7719		3.8115	3.7668
Force			2.6E+6		3.8332	3.7760		3.7593	3.7528		3.7536		

Figure 1: *Both charts illustrate the behaviour of the condition numbers for the HOT* ***B****-matrices corresponding to a (maximal) tensor order of four. In the top chart the results for the HOT hierarchy [2] and in the bottom one those for the single HOT [3] are displayed. The Y-axis measures the (mean) condition number; results are displayed as bars. The corresponding error-bars show the minimum and maximum condition number evaluated for the rotated GES with a given number of encoding directions (X-axis).*

Discussion:

The HOT hierarchy [2] has a more complex **B**-matrix than the single HOT model [3], which explains the generally higher condition numbers for the hierarchy. Regularization methods could be used for solving the linear systems for the hierarchical model [2] more reliably especially in the presence of noise. It was already shown with an example that the reconstruction of complicated fiber structures is possible [2]. In simple cases the hierarchy reduces to the standard DT with an associated low condition number of the **B**-matrix.

References:

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