Comparison of the tensor estimation quality for icosahedral gradient encoding schemes

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Introduction: In diffusion tensor imaging (DTI) the gradient encoding scheme (GES) determines the directional coding of the measured diffusion processes, thereby influencing the quality of the tensor estimation from these measurements. Icosahedral encoding schemes became popular due to their uniformly spread directions, their optimality for measurements with the minimal required number of encoding directions for DTI [1] and their rotationally invariant condition number (more exactly the corresponding estimation matrices' condition number) [2]. The 'optimal' condition number of the icosahedral schemes presented in [2], were found to correspond to a isotropic tensor only [4]. We therefore investigated the usefulness of different icosahedral schemes for DTI evaluations with the expectation that an increase in encoding directions should also increase the tensor estimation quality [2, 3]. The different creation rules of icosahedral schemes (edge-bisection, triangulation, the center of the faces ... [2, 5]) leave room for quality differences.

Methods: The icosahedral GES were evaluated for direction sets with 6, 10, 15, 16, 21, 30, 46, 81, 126 and 321 directions, generated with the above mentioned rules. The fractional anisotropy (FA) values were evaluated for 2061 rotations (azimuth/zenith) of a given tensor. Two types of tensors (cigar-, with eigenvalue relations of [10, 1, 1], FA = 0.87293; oblate-shaped [10, 10, 1], FA = 0.62389) were evaluated to represent the main classes of second order diffusion. Since the estimation of the perfectly isotropic diffusion (ball-shaped [1, 1, 1], FA = 0) is not affected by the orientation of the tensor in the reference coordinate system (relative to the GES) [2, 4] it is not considered here. The measurements were synthesized for several signal-to-noise-ratios (SNR $\in$ {5, 10, 15, 20, 25 and 30}; Gaussian noise distribution) according to a method described in [3], to evaluate the estimation quality for noise-levels similar to the ones in actual measurements. This is necessary since the schemes only differ in the noise-amplification in the tensor calculation. In the absence of noise all reconstructions with the here evaluated schemes are perfect. To reduce random effects in the evaluations the FA-evaluation was repeated 100 times with different noise contributions and the FA results for each tensor rotation were averaged. The mean over all rotations as well as the minimal and maximal value were used in the investigations. For additional classification the minimal, maximal and mean standard deviation (STD) over all 100 evaluations of the rotational dependent FA values were considered.

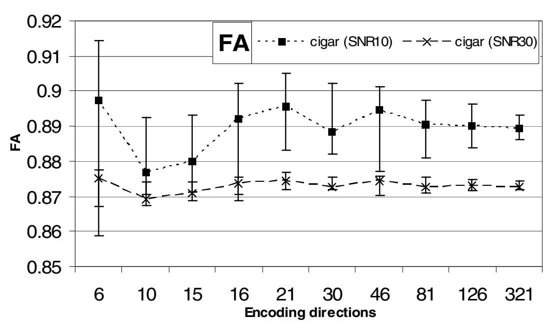
Results: For both tensor types a typical SNR-independent course of the dependence of the FA estimation on the GES could be observed from the SNR-level of 10 onwards. In all our evaluations the interval of the values for both, the FA values and its STD, are strikingly high for the icosahedral scheme with 16 encoding directions. A bit less prominent is the range of values for the scheme with 46 directions. The span of the mean FA values is larger than the one for the scheme with 30 directions for SNR $\geq$ 10. For the cigar shaped tensor it is even larger than the span of the scheme with 21 directions for SNR = {10, 20, 25 and 30}. The span of the STD for the scheme with 46 directions exceeds the one for the GES with 30 directions if SNR $\geq$ 20 in the case of the cigar-shaped tensor and for SNR = {10, 15 and 30} for the oblate-shaped one. These results are exemplarily illustrated in the Fig. 1, on the right.

Discussion and Conclusion: It was observed that the family of icosahedral schemes is not as uniform as their condition number [2] would suggest. With the exception of the schemes with 16 and 46 directions the increase in stability of the tensor estimation with an increase of encoding directions for this GES family could be confirmed. It is advisable to use a relatively high number of encoding directions (greater than 21) to get tensor estimation results that are only little influenced by the orientation of the subject in the measurement reference frame. The behavior of the different groups of icosahedral schemes (dependent on their direction generating rules) needs to be further investigated to determine whether a certain kind of icosahedral data sets is less desirable than others.

The development of the evaluation results over the different SNR levels suggests that one should aim for a minimal SNR around 10 for DTI. Only for SNR $\geq$ 10, the characteristic course is distinguishable.

Previously it was suggested to use the information on the spreading of the direction vectors, which is a measure of equidistribution of the directions, could be used as a classification criterion for GES [5]. The spreading of the icosahedral directions is directly dependent on the creation rule [2, 5] that is used to generate a set with more directions from the regular icosahedron. The inferior spreading of the directions in the GES generated from edge-bisection (15 directions) does not result in inferior tensor estimation. On the contrary, the results are superior to the scheme with 16 wider spread directions. This result suggests that the spreading of the directions holds only limited information for determination of the tensor estimation quality.

One can usually not be certain to have a sample with fibers that are all oriented in the reference frame in a way that provides the rather well behaved mean FA values for the icosahedral GES with 16 and 46 directions. The use of these schemes is therefore not recommended for DTI.



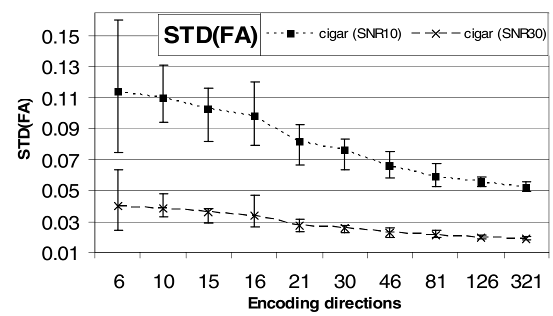


Figure 1: *The evaluation results for the cigar-shaped tensor are plotted for SNR = {10, 30}. The error-bars illustrate the range of values resulting from the evaluation of 2061 tensor rotations.* TOP: *The mean FA values over all rotations. Note the prominent error-bars for the GES with 16 and 46 directions.* BOTTOM: *The mean of the standard deviations of the FA values over 100 evaluations. Note the striking error-bar for the GES with 16 directions. Also the range of results for the scheme with 46 directions is larger than that of the GES with 30 directions for a SNR of 30.*

References:

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