Extension of the Magic Gradient Amplitude Ratio Method Used to Minimize Background Gradient Cross-Terms in Diffusion-Weighted MR

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Introduction

Accurate measurements of the diffusion coefficient with pulsed-field gradient MR are hampered by cross-terms of the diffusion and background gradients caused by inhomogeneities of the magnetic field or susceptibility differences within the sample. Cross-terms of macroscopic background gradients can be compensated by taking the geometric mean of acquisitions with opposite polarity of the diffusion gradients. However, this approach does not suppress cross-terms of microscopic background gradients, i.e. those that vary within a voxel. For spin-echo based diffusion preparation, multiple refocusing RF excitations have been shown to minimize these cross-terms [1]. Cotts *et al* presented a solution for the stimulated echo preparation that is used in samples with a T2 short compared to the required diffusion time [2]. Sun *et al* modified this method to compensate the cross-terms prior to and after the middle interval separately in order to account for diffusing spins that experience different background gradients in the two intervals [3]. However, this solution (Fig. 1a) involves a so-called "magic" amplitude ratio (η) of the applied diffusion-weighting gradients that is at the expense of a reduced diffusion weighting. Here, an extension of this approach is presented that increases the diffusion weighting efficiency.

Methods

The method of Sun *et al* and the extension are shown in Fig. 1. The intervals prior to and after the middle interval (TM) of the stimulated echo preparation are symmetric with respect to TM. The modification involves an additional gradient and maximization of the gradient amplitudes to achieve the strongest diffusion weighting within the giving timing.
Parts of the analytical calculations were performed in Maple (version 9.5.2, Waterloo Maple Inc., Waterloo, Ontario, Canada).

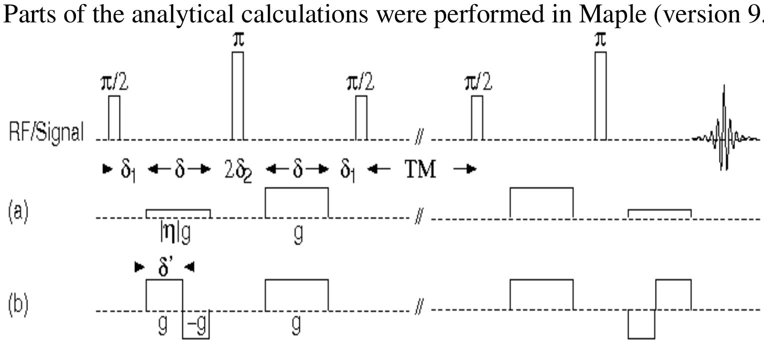

Figure 1: (a) Background gradient compensation scheme of Sun *et al* and (b) extension for increased diffusion-weighting efficiency.

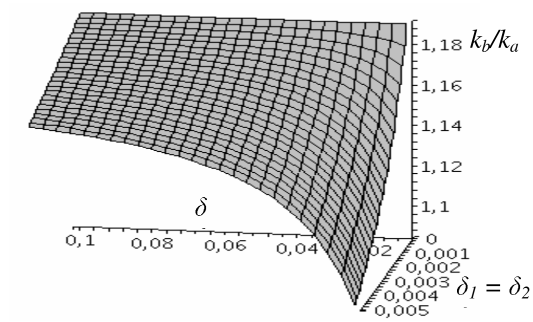

Figure 2: Ratio of k values prior to TM of the extension (Fig. 1b, k_b) and the method of Sun (Fig. 1a, k_a) (δ, δ_1 in ms).

Results

The cross-term b_{cross} within the interval prior to and after TM of the scheme shown in Fig. 1b is given by

$$b_{cross} = g g_b \left[-\frac{1}{3}\delta'^3 - \delta_1 \delta'^2 + (2\delta^2 + 4\delta(\delta_1+\delta_2) + \delta_1^2 + 2\delta_2^2 + 4\delta_1\delta_2)\delta' - \delta^3 - 2\delta^2(\delta_1+\delta_2) - \delta(\delta_1+\delta_2)^2 \right]$$

where g_b is the (unknown) amplitude of the background gradient. The cross-term vanishes for every g_b if

$$\delta' = 2/3\sqrt{18}\sqrt{\delta^2+\delta_1^2+\delta_2^2+2(\delta\delta_1+\delta\delta_2+\delta_1\delta_2)}\,\sin\left[\frac{\pi}{6} - \arccos\left(\frac{12\sqrt{3}(5\delta_1^3+15\delta_1^2\delta+12\delta^2\delta_1+18\delta_1\delta_2\delta+6\delta^2\delta_2+3\delta_2^2\delta+3\delta^3+12\delta_1^2\delta_2+12\delta_2^2\delta_1)}{8\sqrt{6}\sqrt{\delta^2+\delta_1^2+\delta_2^2+2\delta\delta_1+2\delta\delta_2+2\delta_1\delta_2}^{\,3}}\right)\right] - \delta_1$$

according to Cardano's method. Because the *b*-value of the diffusion gradients develops linearly during TM with the gradient integral *k* as the slope, the ratio of the *k* values from the extension and Sun's method has been compared and is plotted in Fig. 2 for typical parameters and assuming $\delta_2 = \delta_1$. The extension reveals *k* values increased by more than 10% with a maximum gain of about 19% ($5/2 - 5\sqrt{2}\sin(\pi/6 - 1/3\arccos(3/8\sqrt{2}))$) obtained for $\delta_1 = \delta_2 = 0$ ($\delta' = 2\sqrt{2}\cos(\pi/3 + 1/3\arccos(3/8\sqrt{2}))\delta \approx 0.52\delta$). Furthermore, the *b* value accumulated prior to TM is more than doubled compared to Sun's method (data not shown).

Conclusion

The extension presented delivers higher *b*-values than the method of Sun *et al* within the given timing.

References

[1] Karlicek RF et al, J Magn Reson 37, 75-91 (1980)
[2] Cotts RM et al, J Magn Reson 83, 252-66 (1989)
[3] Sun PZ et al, J Magn Reson 161, 168-73 (2003)