# Resolving White Matter Fiber Crossings with Diffusional Kurtosis Imaging 

J. H. Jensen ${ }^{1}$, L. Xuan ${ }^{1}$, and J. A. Helpern ${ }^{1}$<br>${ }^{1}$ Radiology, New York University School of Medicine, New York, NY, United States

## Introduction

One of the challenges of fiber tractography is resolving white matter fiber crossings. Since conventional diffusion tensor imaging (DTI) is insufficient for this purpose, several alternative diffusion imaging approaches have been proposed [1]. Among these is $q$-ball imaging which estimates an orientation distribution function (ODF) from high angular resolution diffusion imaging (HARDI) data [2]. Here we show that an approximation for the ODF capable of resolving fiber crossings can also be obtained from the diffusional kurtosis (DK). As a consequence, diffusional kurtosis imaging (DKI), a recently proposed generalization of DTI that yields estimates for the DK [3, 4], may provide information useful for fiber tractography.

## Theory

The ODF in a direction $\hat{\mathbf{n}}(|\hat{\mathbf{n}}|=1)$ is defined by

$$
\begin{equation*}
\psi(\hat{\mathbf{n}}) \equiv \frac{1}{Z} \int_{0}^{\infty} d s P(s \hat{\mathbf{n}}, t) \tag{1}
\end{equation*}
$$

where $P(\mathbf{r}, t)$ is the probability distribution for a diffusion displacement $\mathbf{r}$ in a time $t$ and $Z$ is a normalization constant. The ODF is connected to the diffusional kurtosis by

$$
\begin{equation*}
\psi(\hat{\mathbf{n}}) \approx \psi_{\mathrm{DK}}(\hat{\mathbf{n}}) \equiv \frac{1}{48 \pi^{2} Z t} \int d^{3} u \frac{3+K(\mathbf{u})}{D(\mathbf{u})} \cdot \delta(\hat{\mathbf{n}} \cdot \mathbf{u}) \cdot \delta(|\mathbf{u}|-1), \tag{2}
\end{equation*}
$$

where $K$ is the DK in a direction $u, D$ is the diffusion coefficient in the direction $u$, and $\delta$ is the Dirac delta function. The integral in Eq. (2) can be interpreted as a Funk transform.

## Methods

To test whether the DK approximation for the ODF of Eq. (2) is able to resolve fiber crossings, we consider models consisting of multiple anisotropic, Gaussian compartments [1] and compute the DK-ODF and, as a reference, the exact ODF. Since $P(\mathbf{r}, t)$ can be given analytically for such models, the calculation of the ODFs may be reduced to quadrature, which allows for a straightforward numerical evaluation. In addition, we compute a DT-ODF, based on the diffusion tensor, and a QB-ODF, based on the q-ball imaging method [2]. In all calculations, the normalization constant $Z$ is chosen to be $1 /(8 \pi \bar{D} t)$, where is $\bar{D}$ the mean diffusivity.

## Results and Discussion

Explicit calculations demonstrate that the DK-ODF is capable of resolving crossings with up to four different fiber directions, and an example of a three direction crossing is shown in Fig. 1 with the fiber directions given approximately by the ODF maxima. This is a distinct advantage over the DT-ODF which fails to resolve even double fiber crossings. While q-ball imaging performs similarly in resolving fiber crossings, typical DKI protocols [4] require significantly smaller maximum b-values as well as shorter acquisition times than typical q-ball imaging protocols [2]. Moreover, DKI provides additional diffusion metrics, such as the mean kurtosis,
from the same data set. It should be emphasized that the DK-ODF does not rely on any specific model or extraneous assumptions.

## References

[1] Alexander DC. Ann N Y Acad Sci 2005;1064:113-33. [2] Tuch DS. Magn Reson Med. 2004;52:1358-72. [3] Jensen JH, et al., Magn Reson Med. 2005;53:1432-40. [4] Lu H et al., NMR Biomed. 2006;19:236-47.

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Figure 1. Polar contour plots of ODFs for a model with three fiber directions $(\theta=$ polar angle, $\varphi=$ azimuthal angle). The exact ODF (upper left corner) clearly resolves the three fiber directions (indicated by X's), but the DT-ODF (upper right corner), which can be obtained with conventional DTI, does not. Both the DK-ODF (lower left corner) and the QB-ODF (lower right corner) are able to resolve the fiber directions, illustrating the advantage of these approximations. Only half of the surface of the ODF is shown, since the ODF is symmetric with respect to a reflection through the origin.





