Bayesian analysis of uncertainty in Q-Ball Imaging

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Introduction

Fibre Tracking based on the analysis of diffusion-weighted images has in the last years proven to be an interesting tool to elucidate white matter connections in vivo. The field has recently advanced to techniques in which multiple fibres can be estimated probabilistically [1, 2]. In this study, we will describe a new technique, combining standard Bayesian methods with the existing Q-Ball technique [3].

Theory

A number of authors have suggested the decomposition of the Apparent Diffusion Coefficient (ADC) from High Angular Resolution Imaging (HARDI) data into Spherical Harmonics (SHs) to investigate the presence of multiple fibres in a voxel. Anderson [4] has extended the use of this basis set to express both the diffusion-weighted signal and the Orientation Distribution Function (ODF) derived from a Q-Ball-based analysis of the data. He then proved that there is a direct analytical relationship between the SH-coefficients of the signal ($c_{l,m}$) and the SH-coefficients of the ODF.

In our approach, the SH-coefficients of the signal are taken as a starting point for the probabilistic analysis of the Q-Ball ODF. To this end, we have used the linear regression approach, first proposed in this context by Alexander et al., [5] to estimate $c_{l,m}$. In this approach the spherical harmonics are rearranged in a vector for every applied diffusion-weighted direction. These vectors can be taken together to form a matrix **X**, where each row stands for a different diffusion weighting. This matrix will then be used as a predictor for the signal, given a vector **c** with all elements $c_{l,m.}$ The series is terminated at a predefined maximal order l_{max}.

This linear regression approach can now be extended to a full Bayesian linear regression model for which we need a likelihood and a prior on the parameters to form a posterior pdf. We have chosen to model the noise as Gaussian giving a normal likelihood for the data y, given **c** and the variance of the noise, σ^2. Furthermore we haven chosen a standard noninformative prior on c and σ^2: $p(c, \sigma^2) \sim \sigma^{-2}$. These distributions are conjugate, yielding:

1. A conditional posterior on c, which is Gaussian: $p(c \mid y, \sigma^2) \sim N(\hat{c}, V\sigma^2)$ where $V = (X^T X)^{-}$ and $\hat{c}$ is the standard least-mean squared estimator for c.
2. A marginal posterior on σ^2 given y that is proportional to an inverse-χ^2:distribution.

We can now first obtain samples of σ^2 from this latter distribution and then use these samples to draw samples of c from its own posterior. Then the SH-coefficients are converted to the SH-coefficients of the Q-Ball ODF and used to reconstruct the ODF to obtain its local maxima. We have repeated this procedure 1000 times to obtain a spatial pdf on the directions of the underlying fibres. To assess the feasibility of this procedure we have simulated data from 2 or 3 tensors with equal volume fractions and FA = 0.9 at an angle of 30° relative to each other. The data were simulated using 252 directions at a b-value of 3000 s^2/mm thus simulating a standard Q-Ball experiment. We have simulated different SNR values by adding Gaussian noise to the diffusion-weighted signal. We have calculated the SH-coefficients up to order 8.

Results and Discussion

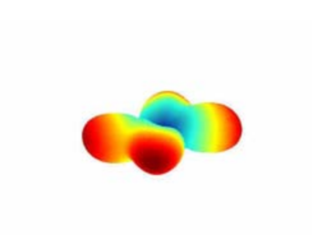

Figure 1. Example of a reconstructed Q-Ball ODF at SNR = 30

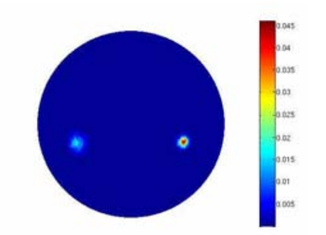

Figure 2. PDF on fibre orientation with SNR = 30

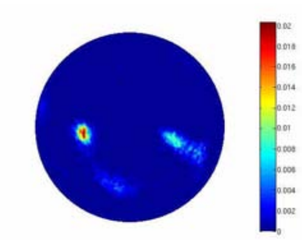

Figure 3. PDF on fibre orientation with SNR = 5

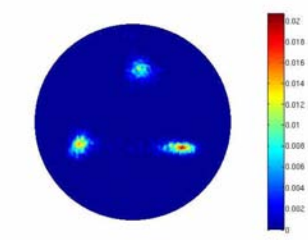

Figure 4. PDF on fibre orientation with 3 tensors at SNR = 5

As can be seen in the figures, the reconstruction of the ODF is successful. In the SNR = 5 case, this results in very sharply peaked pdf's. As SNR decreases the probability is increasingly smeared out and false positives are obtained. Figure 4 shows that our analysis is easily extended to 3 tensors, which is an advantage above the previous methods. We therefore conclude that our approach is a useful extension of the existing methods.

References: 1. Hosey, T., et al., MRM, 2005. **54**(6): p. 1480-1489. 2. Behrens T.E.J., et al., NI, 2006: In Press 3. Tuch, D.S., et al., Neuron 2003 **40:** p. 885-895. 4. Anderson, A.W. MRM, 2005. **54**(5): p. 1194-1206. 5. Alexander, D.C., et al., MRM, 2002. **48**(2): p. 331-340.