Direction dependent overestimation of local wavelengths in MR elastography by discretized Helmholtz inversion

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Introduction: Mechanical properties of human soft tissues can be investigated *in vivo* with dynamic magnetic resonance elastography (MRE) by introducing low frequency shear waves into the body and measuring the resulting deflection field [1]. From the measured deflections physical constants characterizing visco-elastic properties of the tissue can be estimated by inverting the governing equation of motion for local wave numbers or wavelengths. Appropriate simplifications render the general equation of motion invertible [2-7]. One standard method is based on the Helmholtz equation which makes the simplifying assumption of harmonic motion of tissue particles. An analytic solution of these equations for inhomogeneous media with complicated geometries is still not possible. Therefore the Helmholtz equation is discretized and numerically inverted locally assuming constant elastic properties over small regions. In this study the influence of the discretization on the accuracy of results by the Helmholtz inversion process is investigated.

Theory and Methods: The discretized one-dimensional Helmholtz equation of a scalar wave can be written for central differences as

$$k_c^{\,2} u_n + \frac{u_{n+2} - 2u_n + u_{n-2}}{4(\Delta x)^2} = 0,$$

where k_c is the wave number to be calculated, Δx is the equidistant spacing between lattice points and u_n is the deflection at the n^{th} lattice point. If a discretized plane wave is assumed to propagate through the medium

$$u_n = Ae^{i(k_g n\Delta x)},$$

where k_g is the predefined wave number and A is the deflection amplitude, the calculated wave number for all n is

$$k_c = \sqrt{\frac{1-\cos(k_g 2\Delta x)}{2(\Delta x)^2}}.$$

The extension to three dimensions leads to

$$k_c = \left|\vec{k}_c\right| = \sqrt{\frac{1-\cos(k_{x,g} 2\Delta x)}{2(\Delta x)^2} + \frac{1-\cos(k_{y,g} 2\Delta y)}{2(\Delta y)^2} + \frac{1-\cos(k_{z,g} 2\Delta z)}{2(\Delta z)^2}},$$

where $k_{x,g}$, $k_{y,g}$ and $k_{z,g}$ are the components of the predefined wave vector and Δx, Δy and Δz is the equidistant spacing between lattice points in *x*-, *y*- and *z*-direction. The dependency of calculated wave numbers from a predefined wave number (1D) or a predefined wave vector (3D) and the lattice spacing is investigated. For the one-dimensional case the wavelengths $\lambda_c=2\pi/k_c$ were calculated for six predefined wavelengths $\lambda_g=2\pi/k_g$ in dependency of the spacing of lattice points. For the three-dimensional case the wavelengths for a predefined wavelength as well as certain isotropic and anisotropic predefined lattice spacings were calculated dependent of the directions of wave vectors. The ranges of predefined wavelengths and lattice spacings were typical for dynamic MRE. The findings were verified with Helmholtz inversions on simulated plane waves.

Results: Fig. 1 shows the dependency of the calculated wavelength on the lattice spacing for six predefined wavelengths for the one-dimensional case. For non-zero lattice spacing there is always an overestimation of the wavelength which increases to infinity when the ratio between predefined wavelength and spacing reaches 2 (not completely shown). Lower ratios do not fulfill the Nyquist condition and are not considered. The overestimation is below 21% if the ratio is higher than 6. This condition is fulfilled for the shown predefined wavelengths if the lattice spacing is smaller than 1.5 mm.

Figs. 2 and 3 show results for a predefined wavelength of 20 mm in dependency of the direction of the wave vector. In directions along the coordinate axes which correspond to the one-dimensional case, the overestimation is higher than for intermediate directions. For an isotropic lattice spacing the minimum is reached in the direction (1 1 1) in the first quadrant. Due to symmetry the other quadrants show a comparable behavior. For an anisotropic lattice spacing the highest wavelength overestimation results in the direction of the coordinate axis with the largest lattice spacing (Fig. 3). Here the direction with minimal overestimation depends on the degree of anisotropy. Inversions on simulated data resulted in exactly the predicted theoretical values excepting the boundaries.

Discussion: The error introduced in the calculation of local wavelengths by the discretization of the Helmholtz equation was investigated. Effects due to image boundaries were not considered. The analysis of the error for plane waves showed that the discretization leads always to an overestimation which is also dependent on the direction of the wave vector of the considered plane wave. To get an estimation error of less than 21 %, the driving frequency in MRE should be adapted so that the ratio between the expected wavelengths and the used lattice spacing in each spatial direction is greater than 6. This condition is usually fulfilled in MRE images for wavelengths of several centimeters and a typical in-plane resolution of 1-2 mm. However, often the slice thickness is chosen to be 4-6 mm due to sequence restrictions or to get a good SNR. If a comparable distance is also used for the out-of-plane lattice spacing in 3D data, this may lead to a severe wavelength overestimation. A similar guideline is probably also reasonable for discretized inversions of other equations of motion.

References:

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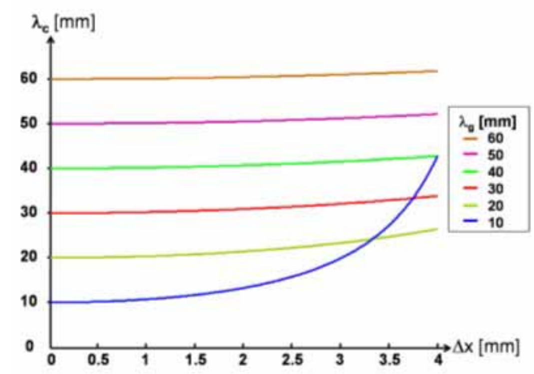

Fig. 1: Calculated wavelengths λ_c for six predefined wavelengths λ_g (10 to 60 mm, 10 mm increments) versus lattice spacing. Different line colours correspond to different predefined wavelengths.

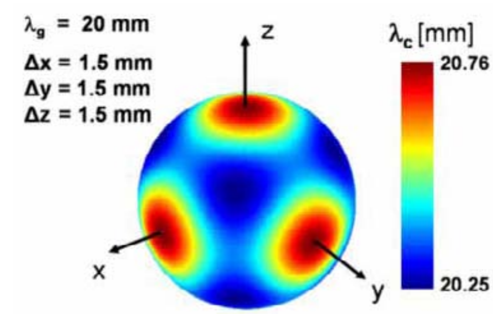

Fig. 2: Calculated wavelengths λ_c for a predefined wavelength λ_g of 20 mm and an isotropic lattice spacing Δx, Δy and Δz of 1.5/1.5/1.5 mm. Wavelengths are displayed color-coded on a sphere to visualize the direction dependency.

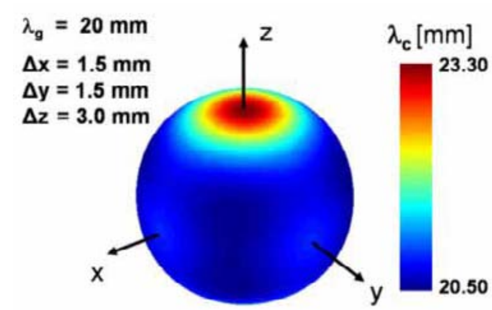

Fig. 3: Calculated wavelengths λ_c for a predefined wavelength λ_g of 20 mm and an anisotropic lattice spacing Δx, Δy and Δz of 1.5/1.5/3.0 mm. Wavelengths are displayed color-coded on a sphere to visualize the direction dependency.