An inverse method for designing RF phased arrays with optimal coil geometry

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Introduction: Radio-frequency phased arrays are comprised of many, closely spaced coils covering a large volume. This affords the coil array the SNR and resolution of a surface coil, but over a field of view normally associated with a volume coil [1]. Many established coil geometries exist for phased arrays, however they typically involve combinations of simple loop or rectangular coils or straight rods [2]. Coil design techniques may then look to optimise the amplitudes and phases of the currents driving each coil (*eg* [3]), or optimise the positions and sizes of the coil elements (*eg* [4]). These techniques may also be developed specifically for the application of parallel imaging. In the current work, focus is directed towards appropriate geometries of the individual coils for a variety of phased array cases, rather than drive schemes or image reconstruction techniques. Finite length RF coils that induce highly homogeneous magnetic fields have previously been obtained using a target-field type design method [5]. A similar inverse method is presented here for the theoretical design of RF phased array coils, covering a wide variety of array sizes and field focussing and polarisation design considerations.

Method: At high-field, a time-harmonic solution to Maxwell's equations is required to accurately describe the electromagnetic fields. This can be achieved through the use of Green's functions and yields the following expression for the magnetic field induced by some arbitrary surface current density:

$$\overline{\boldsymbol{H}}(\boldsymbol{r}) = \frac{1}{2\pi}\iint_S e^{-i\alpha R}\left(\frac{i\alpha}{R^2}+\frac{1}{R^3}\right)(\boldsymbol{r}'-\boldsymbol{r})\times\bar{\boldsymbol{j}}(\boldsymbol{r}')\,dA'. \tag{1}$$

The form of the current density vector $\boldsymbol{j}$ must be chosen carefully to represent a phased array coil. Assuming a current density that exists on the surface of a cylinder of radius a and length $2L$, this cylinder is broken up into KxQ subregions (in θ and z respectively), with independent current density occurring in each subregion. The components of the current density in any particular subregion are related to one another via the continuity equation, and for each (k,q) subregion are chosen to be expressed as Fourier series in θ and z, ensuring that zero current density exists on all boundaries. The aim then is to specify some target magnetic field in equation (1) over the surface of a target region (DSV) and solve for the unknown Fourier coefficients c_{mn}^{kq} of the current density components, for all the KxQ subregions. This makes equation (1) a Fredholm integral equation of the first kind and therefore highly ill-conditioned. A regularisation strategy is implemented to overcome this problem, in which the error between induced and target magnetic fields over the surface of the DSV is minimised along with a smoothness penalty function related to the curvature of the coil windings over the entire cylinder. The impact of the penalty function depends on the regularising parameter λ, whose value is open for experimentation.

Results: The design method allows any array size to be considered simply by altering the values of K and Q. The smoothness of the coil windings can be improved simply by increasing the value of the regularising parameter λ, at the expense of field homogeneity. The location and size of the DSV can be varied to consider coils specifically designed to focus the RF field to particular locations within the coil volume. The effects of field polarity can also be investigated by careful specification of the target field. This includes the consideration of quadrature coils inducing circularly polarised magnetic fields. For all cases considered, improved magnetic field homogeneity is observed throughout the DSV as array size is increased. This is true for both symmetrically and asymmetrically located DSVs. As expected, the level of homogeneity attainable decreases as the DSV is moved further away from coil centre. However, improvement in field homogeneity with increased array size is even more pronounced in these asymmetric cases. For radially asymmetrically located DSVs (moved away from the central coil axis) the direction of polarisation of the assumed target field becomes important. This alters the winding pattern solution for the phased array coil considerably and also affects the attainable homogeneity of the induced magnetic field. As an example of a typical set of results using this design method, figure 1 displays the coil windings ($\lambda = 10^{-16}$) for a 4x3 array as they would be wrapped around the coil cylinder, assuming a DSV (shown) of radius a = 0.05 m, centred at the point $(r,\theta,z) = (0.05,\pi/2,0)$, and a y-polarised target field. At low operating frequencies (*eg* 64 MHz) each set of nested concentric coils would be replaced by a spiral and driven together, however at higher frequencies (*eg* 128 MHz and above) the concentric rings must be driven individually to avoid current phase retardation effects. Figure 2 displays a 5% contour plot of the correspondingly induced magnetic field on the (r,z) plane at $\theta = \pi/2$, with the boundary of the DSV represented by the dashed line. Figure 3 displays a plot of this magnetic field through the centre of the DSV (vertical dashed lines), along a line of constant r = 0.05 m and $\theta = \pi/2$. Figures 2 and 3 show that the coil array in figure 1 is capable of inducing a highly homogeneous field throughout the DSV.

Conclusion: A robust design method for RF phased arrays has been presented that allows the investigation of optimal geometry and position of the individual coils. This represents a departure from the use of traditional phased array coil designs and produces novel winding patterns capable of inducing highly homogeneous magnetic fields. The described method represents a foundation to future work in which the effects of loading and inductive coupling between coil elements may be considered. The appeal of the method is its versatility in terms of design considerations, including array size, field focussing and polarisation, and the ability to trade-off coil winding simplicity with field homogeneity.

References:
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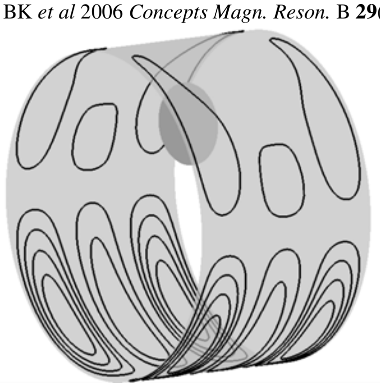

Fig 1. Coil windings for a 4x3 array with a radially asymmetrically located DSV (shown).

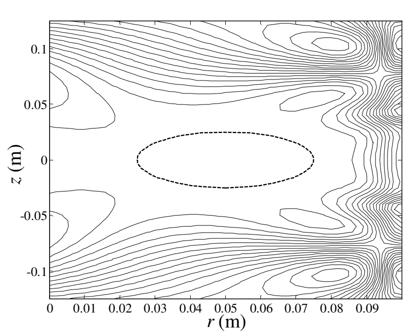

Fig 2. Contour plot of the induced Hy field on the plane $\theta = \pi/2$ (DSV dashed line).

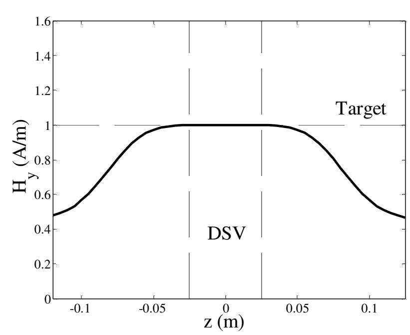

Fig 3. Plot of the induced Hy field on the line r = 0.05 m and $\theta = \pi/2$.