Boundary Element Method for Calculation of Induced Electric Fields due to Swiched Magnetic Field Gradients and Movement in Strong Static Fields

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Introduction MRI depends on the use of both high static magnetic fields and rapidly switched magnetic field gradients. Natural movements in the large static field cause induced currents in conducting tissues and have potential bio-effects such as dizziness and metallic taste on the tongue. The switching of the gradients may produce PNS (peripheral nerve stimulation) in subjects. It is not feasible to measure the induced current or electric field in an object directly. Hence for complex geometries we are forced to use numerical analysis techniques such as the finite difference time domain (FDTD) method[1] or impedance method[2]. Boundary element methods (BEM) have already been applied to electromagnetic field calculations for analysis of magneto- and electro-encephalography (MEG/EEG)[3] data and have been shown to provide an excellent framework for low frequency studies. However, these methods have not previously been applied to the calculation of induced currents in the human body due to exposure to time varying magnetic field gradients or movement in and around an MRI scanner. Here a BEM approach suitable for such studies has been developed and tested by comparison with analytic solutions[4] for simply shaped objects exposed to switched gradients. The BEM approach has also been used to evaluate the induced electric fields and current densities resulting from rotation of a head model in the bore of a 7T magnet.

Theory Gradient switching frequencies for an MR scanner are usually below 10 kHz. Natural movements of the body lie in the range 0 – 20 Hz. At these low frequencies, the electromagnetic properties of the body allow us to use a quasi-static approximation. The electric field induced in the body can be expressed in terms of the scalar (V) and vector potentials (**A**), given by $\mathbf{E} = -\nabla V - \partial\mathbf{A}/\partial t$ **(1)**. The computation of the time dependent second term is straightforward as **A** is produced by the current flowing in the coil (main magnet or gradient); whereas the conservative term (generated by the charges accumulated at the boundaries between domains of different conductivity, σ) can not be simply evaluated. If the body is considered as a multi-compartment volume conductor made of different tissues or domains (Ω_k) with homogeneous electromagnetic properties, then we find that V satisfies Laplace's equation, $\nabla^2 V = 0$, within every domain. By using the free-space Green's function ($\phi^{'}$) of this partial differential equation an integral equation for the potential V at every domain boundary can be identified[5]. The application of the continuity of the current flowing at every boundary between regions (S_i) yields the final integral representation for the potential in every domain in terms of the potentials and their derivatives over all the boundaries. Using **Eq.1**, an integral expression for **E** in terms of the potentials and their derivatives can be formed **(2)**

$$\mathbf{E}^k(\mathbf{x}) = -\frac{\partial\mathbf{A}(\mathbf{x})}{\partial t} - \sum_{i=1}^{n}\left(\frac{\sigma^{i+1}-\sigma^i}{\sigma^k}\right)\int_{S_i}\left[\frac{\partial A^n(\mathbf{y})}{\partial t}\nabla\phi^*(\mathbf{x},\mathbf{y}) + \nabla\frac{\partial\phi^*}{\partial n}(\mathbf{x},\mathbf{y})V^i(\mathbf{y})\right]ds$$

$$\mathbf{x}\in\Omega_k, \mathbf{y}\in S_i$$

Method: Theoretical[4] expressions for **A** have used for evaluation of the effect of switched magnetic field gradients. An analytic form of **A** has also been used to generate an appropriate form of $-\partial\mathbf{A}/\partial t$ for object translations in a uniform 7 T magnetic field [1]. Flat (linear) triangle meshes for the surface of simple geometries (spheres and cylinders) were generated numerically. For more complex surfaces such as human organs (brain and head), data from the HUGO human body model (Medical VR Studio, GmbH, Lorrach, Germany) was used. By considering a uniform (constant BEM) value of V in each of the elements, a system of equations is formed from the set of integral equations whose solution is V. **E** can subsequently be found from V and $\partial\mathbf{A}/\partial t$.

Results: BEM results for the electric field induced in a uniform spherical conductor by switched magnetic field gradients are found to be in good agreement with theoretical values [4]. Figure 1 shows the theoretical and numerical values of the modulus of the electric field at each mesh element, for a sphere of radius 0.1 m exposed to a z-gradient of 30 mTm^{-1} amplitude varying sinusoidally at 1 kHz. The centre of the sphere was shifted 0.1 m axially from the origin of the gradient. For a spherical surface meshed using 1500 elements, the calculations took 3 minutes to run on a PC dual Pentium III motherboard 2x850 MHz. Figure 2 shows the electric field (in Vm^{-1}) found at the surface of the sphere using the BEM. Figure 3 shows the current density (in Am^{-2}) induced in the central zx-plane of a set of concentric spheres of radii 0.05, 0.10 and 0.15 m and conductivities 1, 0.0125 and 1 Sm^{-1} forming a simple model of the brain, skull and scalp exposed to an x-gradient (30 mTm^{-1} varying at 1kHz). Figure 4 shows the current density for the same sphere rotating about the y-axis in a static 7T magnetic field with an angular velocity of 1 rads^{-1}. Figure 5 shows the induced electric field at the surface of the brain for the same time-varying x-gradient as used in Figure 3 for a two-compartment head-brain (conductivities 1 and 10) model. A computation time of 60 minutes was required for the 3-sphere model which used 4500 elements in total.

Discussion BE methods have distinct advantages over finite element methods for problems in the quasi-static regime. They are also particularly suited to geometries with variable resolution i.e. small domains within large volumes. The techniques reported here can be applied to modeling of the currents induced in tissue at low frequencies and should be valuable in the study of bio effects due to high static and switched magnetic fields. For large numbers of mesh elements the computational time can be reduced by employing either the domain decomposition method (DDM) or the fast multi-pole method (FMM).

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