Modeling Loaded RF Coils using the Method of Moments

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INTRODUCTION

The interaction between biological tissues and the electromagnetic field produced by the RF coil necessitates the need to develop complete full wave analysis methods. Several numerical methods are available for modeling coil-tissue interactions by solving the full set of Maxwell's equation [1]. The Method of Moments (MoM) has been used extensively for modeling unloaded RF coil structures, but it suffers from increased implementation complexity and is more prone to numerical errors when biological loads are introduced [1]. In the present study, we discuss a new approach based on the MoM that combines two sets of basis functions.

THEORY

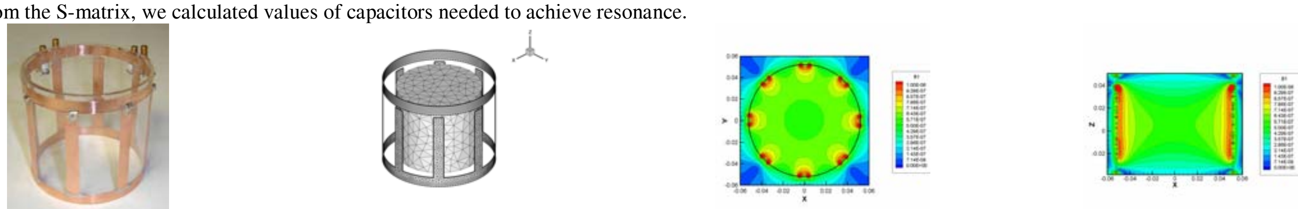
Our approach to formulating a MoM model combines the Rao-Wilton-Glisson (RWG) [2] basis function and the divergence-free solenoidal basis function described in [3]. The RF coil is discretized into triangular surface elements while the biological body of interest is discretized into tetrahedral volume elements. The RWG basis function is associated with each interior edge of the triangular domain where it is used to describe the surface current density on the surface of the RF coil. For the case of the biological body, the divergence-free solenoidal basis function is used to describe a new vector quantity that represents the linear combination of the volume current density and the displacement current density [4]. This new vector quantity is termed the total current density and is always divergence-free as dictated by the continuity equation irrespective of the electrical properties of the biological body. Thus, spurious charges which are the main source of numerical problems in MoM volume formulations are not generated [2]. The electric field integral equation (EFIE) for our MoM implementation can conveniently be written in mixed potential form as

$$\mathbf{E}(\mathbf{r}) = -j\omega\mu_0\left[\int_{Surface}\mathbf{J}_S(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,dS + \int_{Volume}\mathbf{J}_V(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,dV\right] + \frac{1}{j\omega\varepsilon_0}\nabla\left[\int_{Surface}\left(\nabla'\cdot\mathbf{J}_S(\mathbf{r}')\right)G(\mathbf{r},\mathbf{r}')\,dS + \int_{Volume}\left(\nabla'\cdot\mathbf{J}_V(\mathbf{r}')\right)G(\mathbf{r},\mathbf{r}')\,dV\right] \quad (1)$$

The surface current density is given by $\mathbf{J}_S(\mathbf{r})$ while the equivalent volumetric current density and the free space Green's function are given by $\mathbf{J}_V(\mathbf{r})$ and $G(\mathbf{r}, \mathbf{r}')$ respectively. The equivalent volumetric current density is a scalar multiple of the total current density as shown in [4]. The coupled MoM equations are obtained using a Galerkin-type enforcement condition in which the basis and testing functions are identical. From the solution of the coupled MoM equations, we can obtain all other pertinent electromagnetic parameters of the coil-body system.

SIMULATION METHOD

For a low-pass quadrature birdcage coil, Fig. 1(a), a model was generated as shown in Figure 1(b) with diameter of 104 mm and height of 86 mm. The model had 8 rungs each of width 9.5 mm. The biological load was a simple cylinder of diameter 84 mm with electrical properties of human muscle at 63.6 MHz. The birdcage model was discretized into 6038 triangular elements, while the cylindrical load was discretized into 3452 tetrahedra. The birdcage model was treated as an 8-port system where each port corresponds to a capacitor location. The MoM formulation is next used to determine the scattering matrix (S-matrix) of the coil model at 63.6 MHz. From the S-matrix, we calculated values of capacitors needed to achieve resonance.

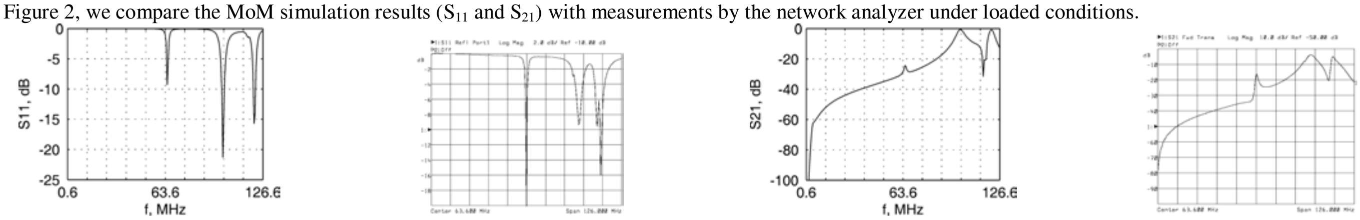


(a) (b) (c) (d)

Figure 1: Low-pass birdcage coil. (a) photograph, (b) model, (c) B_1 field plot in *xy*-plane, (d) B_1 field plot in *xz*-plane.

RESULTS AND DISCUSSION

In order to verify our MoM formulation, we built a prototype of a quadrature birdcage coil with dimensions identical to those of our coil model (see Figure 1(a)). In Figure 2, we compare the MoM simulation results (S_{11} and S_{21}) with measurements by the network analyzer under loaded conditions.



(a) (b) (c) (d)

Figure 2: S_{11} and S_{21} plots. (a) simulation of S_{11}, (b) measurement of S_{11}, (a) simulation of S_{21}, (b) measurement of S_{21}.

We found that the capacitors we calculated made the structure resonate at a frequency very close to 63.6 MHz. By slightly adjusting the capacitors' values we were able to tune and match both channels to the load. In addition, our method allows us to extrapolate the S-matrix to neighboring frequencies. We observe that shapes of the S_{11} and S_{21} curves are very similar in simulations and in measurements. There is, however, a noticeable disagreement between values of additional (high-frequency) resonances. This can be explained by the fact that the extrapolation method only works well at frequencies close to the center frequency (63.6 MHz in this case).

CONCLUSION

We demonstrated that the MoM can be efficiently used to simulate the frequency behavior of loaded MRI RF coils. Using the example of the quadrature birdcage coil, we calculated capacitor values that are necessary to make the coil resonate at a specified frequency. By building and testing the prototype of the coil, we verified the accuracy of our predictions and studied the behavior of the coil at neighboring frequencies.

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