Efficient RF Coil Simulations with Curvilinear Quadrilateral-Element Method of Moments

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Introduction: Surface integral-equation based MoM is an effective tool for full-wave simulations of high-field RF coils (1). It seeks solutions of equivalent currents on the surface of conductors and/or dielectric bodies. In many MRI applications, the geometrical features of RF coils are much smaller than that required to resolve current distributions on coils if coil loss is not the major concern, e.g., SNR or g-Factor maps in sample. To avoid excessive large number of unknowns for geometrical modeling, we applied curvilinear quadrilateral-elements MoM. This approach allows modeling curved structures within one single element, instead of using multiple low-order elements for geometric modeling per se.

Methods: A quadrilateral surface can be modeled by two parametric curvilinear coordinates *(u,v)* through a set of shape functions as $\vec{r} = \vec{r}(u,v)$. A common choice of shape functions is the Cartesian product of Lagrange polynomials, which can be of arbitrary orders to model complex geometries. Based on the parametric description, the co-variant $\vec{a}_u = \frac{\partial \vec{r}}{\partial u}, \vec{a}_v = \frac{\partial \vec{r}}{\partial v}$, and contra-variant $\vec{a}^u = \nabla u, \vec{a}^v = \nabla v$ unitary vectors can be calculated (Fig. 1).

The covariant unitary vectors are tangential to *u*- and *v*- directed edges and the contra-variant unitary vectors are normal to them. The surface current is then expanded inside each element in the two co-variant direction as $\vec{J} = J_u \vec{a}_u + J_v \vec{a}_v$. Due to the relationship of $\vec{a}_u \cdot \vec{a}^u = \vec{a}_v \cdot \vec{a}^v = 1$, the co-variant components of the surface current are normal to the *u*- and *v*- directed edges. This property is desired in modeling current flowing across element boundaries. Each co-variant components can be further expanded with a set of hierarchical higher-order polynomials as, for instance, the *u*-component (2),

$$J_u = \frac{1}{J_s(u,v)} \sum_m^M \sum_n^N a_{mn}^u P_m^u P_n^u$$

where $J_s(u,v)$ is the surface Jacobian and P_m^u and P_n^u are hierarchical polynomials. For the problem at hand, the lowest order polynomials, i.e., the linear ones, are sufficient.

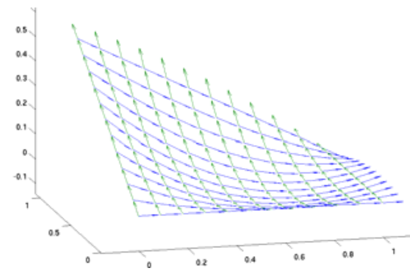

Figure 1: The co-variant unitary vectors on a bi-quadratic quadrilateral surface

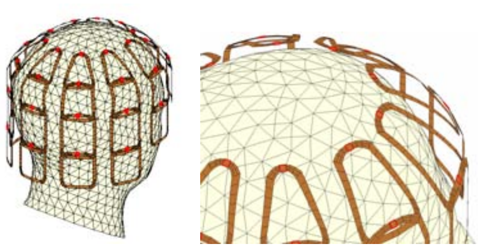

Figure 2: Left: the MoM model of a 32-channle receive coil with the human head phantom. Right: details of the modeling. The two 3X3 meshes on top of each coil correspond to bi-quadratic elements.

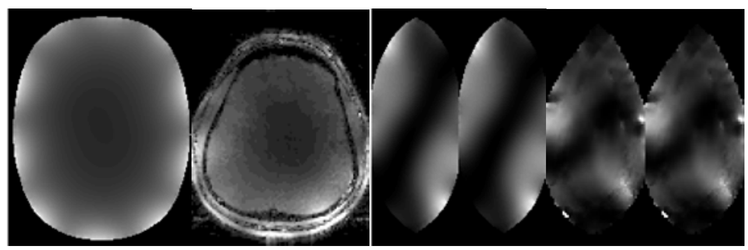

Figure 3: Left: the simulated (far-left) and the measured (mid-left) phase-sensitive combined SNR maps. Right: the simulated (mid-right, max=1.07) and the measured (far-right, max=1.1) rate-2 g-factor maps.

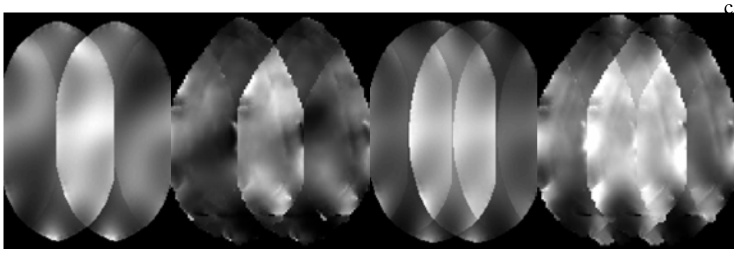

Figure 4: Left: the simulated (far-left, max=1.22) and the measured (mid-left, max=1.4) rate-3 g-Factor maps. Right, the simulated (mid-right, max=2.05) and the measured (far-right, max=2.0) rate-4 g-factor maps.

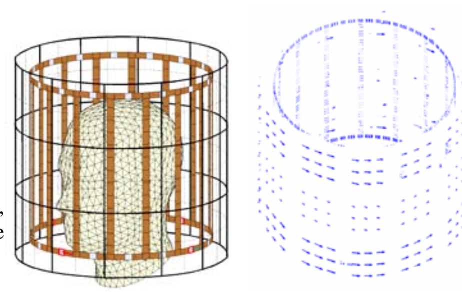

Figure 5. Left: the MoM model of a 32-rung shielded birdcage coil. Right: the simulated current distribution on the shield.

Results and Discussion: An in-house computer program was implemented with the standard ANSI C++ language and compiled on Linux system. We first applied this method to a 32-channel receive-only coil array for brain parallel imaging at 7.0 Tesla. Due to the much shorter wavelength in phantom and the lack of quadrilateral model of the head, only coils are modeled with quadrilateral elements. Bi-quadratic curvilinear elements are used to model the upper and lower curvature of each coil. To study the performance of the coil array, each coil element was simulated individually and the sensitivity profiles are combined afterward. On average, there were 6900 unknowns in each individual simulation, with about 20 of them associated with a coil. Each simulation requires about 700 MB memory (with double precision numbers). It took about 150 seconds on a 2.6 GHz AMD Opteron 252 processor. The entire simulation finished within 80 minutes. The results are shown in Figs. 2 and 3 together with the experiment results. In general, the simulation matches the experiment well. We further applied this method to simulate the current distribution on the shield of a shielded 7.0 Tesla birdcage coil. The model and the instantaneous current distribution are shown in Fig. 5, where the shield is modeled with bi-quadratic elements and the lowest order basis functions.

Conclusion: We developed a higher-order surface integral-equation approach for simulating RF coils. Numerical results show its ability in geometric modeling and efficiency. This method is useful in general RF coil design at high-field.

References: 1) R.F. Harrington Field computation by method of moments. IEEE Press. 2) M. Djordjevic, et al, IEEE-TAP 52: p2118-2129, 2004.