A Simple Noniterative Principal Component Technique for Rapid Noise Reduction in Parallel MR Images

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Introduction: The scan time reductions afforded by parallel imaging are well known to come at the cost of noise amplifications (characterized by the g-factor) that can corrupt image quality, essentially limiting maximal practical acceleration. Recently, a promising method was proposed for reducing g-factor-related noise [1] by taking advantage of the fact that at acceleration factors approaching the number of coils, the parallel image reconstruction matrix tends to be dominated by one singular value and vector. Larkman et al suggested searching for a multiple of this dominant singular vector which, when subtracted from the image, minimized joint entropy (JE) between the accelerated image and a reference image. This algorithm is attractive because its capabilities improve as acceleration factor increases, but it is computationally intensive. We propose a method that uses a similar principal-component approach, but eliminates the need for a search, producing results similar to the JE approach in a single easily-computed step. This simple algorithm, which may be executed in real time as images are acquired and reconstructed, should allow significant reductions in g-factor-related noise for highly accelerated scans with the aid of a reference image of either similar or different contrast.

Theory: Cartesian SENSE image reconstruction involves inversion of an encoding matrix C composed of complex coil sensitivities at each set of aliased positions in the target image. C can be factorized using a singular value decomposition (SVD) as C = USV^{+}. Generalizing the Larkman et al notation in a bra-ket formulation, the inverse matrix, or reconstruction matrix, can be written as $C_{inv} = VS^{-1}U^{+} = \Sigma_k |v_k\rangle s_k^{-1}\langle u_k|$, where $|v_k\rangle$ and $\langle u_k|$ are columns and rows of the V and U^{+} matrices, respectively, s_k is the k-th diagonal element of the diagonal S matrix, and the sum over k runs from 1 to the acceleration factor. If the unfolded pixels are represented in a complex vector $|X\rangle = |X'\rangle + |dX\rangle$ with true pixel intensity $|X'\rangle$ and noise contribution $|dX\rangle$, and the folded pixels are represented in a complex vector $|S\rangle = |S'\rangle + |dS\rangle$ once again separating true pixel intensity $|S'\rangle$ from noise contribution $|dS\rangle$, then the SENSE-reconstructed image (complete with amplified noise) is $|X\rangle = C_{inv}|S\rangle$ and the true noise-free image intensity is $|X'\rangle = |X\rangle - |dX\rangle = |X\rangle - C_{inv}|dS\rangle = |X\rangle - \sum_k |v_k\rangle s_k^{-1}\langle u_k|dS\rangle \equiv |X\rangle - \sum_k |v_k\rangle s_k^{-1}\delta S_k$. Here, δS_k is an unknown scalar value defined by the inner product of k-th vector $\langle u_k|$ with the unknown noise vector $|dS\rangle$. At high accelerations, C becomes ill conditioned and, as a result, C_{inv} is dominated by the first few singular values and singular vectors. Thus, only one or a small number of complex quantities δS_k need be found, and the appropriate multiples of δS_k with the corresponding singular values and vectors subtracted from the noisy reconstructed image, to estimate the noise-free image intensities. The technique proposed by Larkman et al uses an exhaustive search to find the value of a single dominant δS_k which minimizes joint entropy between the resultant image and a reference image. We propose that algebraically solving for δS_k using a simple least-squares fit to a reference image can yield results similar to those of the JE search, but in a fast and computationally efficient manner. In particular, for one dominant singular value, we can write $|dX\rangle = |v_1\rangle s_1^{-1}\delta S_1 \approx |X\rangle - |X_{ref}\rangle$ and, using the usual Moore-Penrose least squares solution, $\delta S_1 \approx s_1 \left(|v_1\rangle\langle v_1|\right)^{-1}\langle v_1|\left(|X\rangle - |X_{ref}\rangle\right)$. Generalization to multiple values of δS_k is trivial, with $|v_1\rangle$ being replaced by a matrix containing a larger subset of the columns of V.

Methods: The method was tested in simulations using T1- and T2-weighted brain images with and without lesions, as well as in phantom acquisitions using 8- and 32-element arrays at 1.5T. Various accelerations were tested by decimating fully-sampled data. All reference images were low-resolution images of the sort which are typically acquired as coil sensitivity references in a clinical setting. After a SENSE reconstruction was performed, we either calculated δS_k for one or more singular values/vectors, or else used an implementation of the Larkman et al procedure, searching in the complex plane for a single δS_1 that minimized JE. Results were assessed by: a) visual comparison with the gold standard unaccelerated image, b) calculation of modified g-factor maps, and c) subtraction from the gold standard.

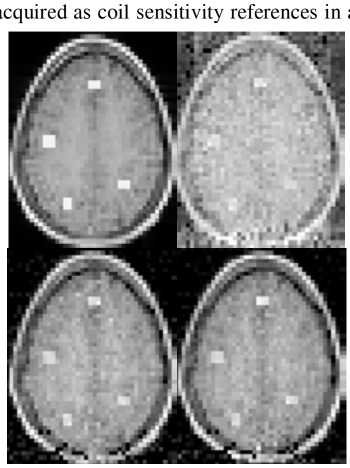

Fig. 1: Simulated brain images. Upper left: Unaccelerated gold standard. Upper right: 4x SENSE reconstruction. Lower left: JE noise reduction. Lower right: Least-squares noise reduction.

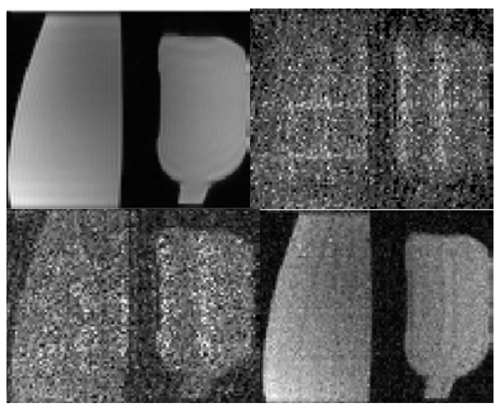

Fig.2: Phantom data obtained with a 32-element (4 x 4 x 2) body array [2]. Upper left: Unaccelerated gold standard. Upper right: 32x (8x4) SENSE reconstruction. Lower left: 1-component least squares noise reduction. Lower right: 9-component least-squares noise reduction.

Results: In all trials where reference and target image were of similar contrast, our method of calculating δS_k greatly reduced noise in the SENSE image. Discrete lesions present in the simulated accelerated brain images but not in the references were seen to be preserved. Our search-free method did not produce significant artifacts in noise-free simulations. In general, more effective noise reduction was observed for larger numbers of δS_k values used, as is to be expected from theory. JE reconstruction time was approximately 10 hours in our implementation, as compared with approximately 10 seconds for our least-squares estimation technique. Indeed, due to the dramatically increased computational burden for a multidimensional search, the JE method is currently practical only for determination of a single δS_1. When only a single value δS_1 was used for each set of aliased pixels, the value determined in the JE search was generally very nearly equal to the value determined by least squares fitting.

Discussion and Conclusions: By calculating δS_k from a SENSE-reconstructed image and low-resolution reference image, we are able to significantly reduce g-factor-related noise in highly accelerated scans. Although the method relies on prior information in the form of a reference image, the algorithm remains substantially protected against simple replication of reference image content, since aliased sets of voxels may only be changed in fixed ratios as defined by the singular vectors $|v_k\rangle$. This prevents unrestricted modification of any given pixel value to match prior information. Noise-reduction using our least-squares approach is on the order of 3 orders of magnitude faster than for the algorithmically difficult JE approach, which yields similar results for a single δS_1 value. Moreover, the least-squares approach allows incorporation of the contributions of multiple singular values/vectors for each set of aliased voxels, with the important caveat that, as more components are used, the result is less effectively protected against replication of prior information from the reference image. The least-squares method works best when target and reference images are of same contrast, and it has at present only been applied for regular Cartesian undersampling, though the algorithm is efficient enough that other sampling trajectories might be considered. Overall, the approach proposed here is a promising candidate for rapid noise reduction in highly-accelerated images.

References: 1. Larkman et al, MRM, 2006; 55(1):153-160. 2. Zhu et al, MRM, 2004; 52(4):869-77.