Data Acquisition Considerations for Compressed Sensing in MRI

J. P. Haldar[1], D. Hernando[1], B. P. Sutton[2], and Z-P. Liang[1]

[1]Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, United States, [2]Bioengineering, University of Illinois at Urbana-Champaign, Urbana, IL, United States

INTRODUCTION

Compressed sensing (CS) theory [1-3] provides a methodology with which compressible signals, such as MR images, can often be recovered using a small of measurements. This result has been applied successfully to significantly accelerate various MRI experiments by acquiring only a small set of quasi-randomly selected *k*-space measurements [4,5]. However, the specific choice of data acquisition scheme will affect the quality of the reconstruction in the CS framework. In this work, we provide a systematic study of different data acquisition schemes for CS-MRI.

THEORY

The imaging equation in MR can be modeled as $\mathbf{d}=\mathbf{E}\boldsymbol{\rho}+\boldsymbol{\eta}$, where $\mathbf{d}$ is the measured data, $\boldsymbol{\rho}$ is the desired image, $\mathbf{E}$ is the encoding matrix, and $\boldsymbol{\eta}$ is a noise vector. CS provides theoretical bounds on the reconstruction error in terms of the compressibility of $\boldsymbol{\rho}$ and characteristics of $\mathbf{E}$ and $\boldsymbol{\eta}$ [2], and shows that this reconstruction error can be small even for a very small number of measurements. However, the quality of these reconstructions requires that $\mathbf{E}$ obeys a *uniform uncertainty principle* (UUP), which loosely translates to the condition that the observations resulting from any sufficiently sparse $\boldsymbol{\rho}$ will not have concentrated energy [3]. The spirit of the UUP is not satisfied by Fourier measurements for most MR images, since signal energy for these images is typically concentrated in central *k*-space. However, it has been shown that more general randomized measurement schemes are well-suited to CS reconstruction [2], and thus might be a better choice for CS in MRI. Since MRI has the flexibility to implement various non-Fourier encoding schemes [6], a natural and important question is which types of practical data acquisition schemes are best for CS reconstruction. In this work, we compare randomized Fourier encoding, non-Fourier encoding and more traditional acquisition and reconstruction methodologies, and also investigate the reliability of the reconstructions in the presence of data noise.

RESULTS AND DISCUSSION

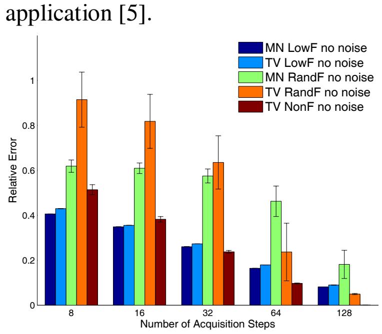
We considered the following data acquisition schemes: traditional low-frequency Fourier (LowF) measurements, randomized Fourier (RandF) measurements (weighted so that low-frequency samples were more common [4]), and non-Fourier (NonF) measurements, where instead of phase encoding, the encoding function is a random vector. Simulations were performed using MR brain images, and the resulting reconstruction errors are shown for different numbers of acquisition steps for both a noiseless case and a noisy case in Figs. 1 and 2, respectively. Reconstructions were performed using both standard minimum norm (MN) reconstruction and an ℓ_1 approximation to total-variation (TV) reconstruction [7].

The simulation results indicate that non-Fourier encoding outperforms RandF and LowF encoding in the noiseless case given sufficient data. However, as the number of data samples becomes very small, traditional sampling schemes outperform schemes with data acquisition tailored for CS. In the noisy case, things are much different, since RandF and NonF necessarily sacrifice SNR compared to LowF sampling, due to the concentrated energy at low-frequency *k*-space. The better performance of RandF versus NonF for a larger number of data samples in this case is due to the specific randomization pattern used for RandF, which favored the higher-energy low-frequency samples.

These simulations reveal additional interesting behavior; CS reconstructions using RandF (primarily) or NonF (to a lesser extent) can appear to be very high resolution, while failing to accurately reconstruct large image features when the number of data points is too small. This is in contrast to LowF sampling, in which case the reconstructions never appear significantly higher resolution than would be expected based on the data collection, but also don't miss reconstructing large scale features. This is an important point: for standard acquisition and reconstruction approaches, the resolution and noise properties are very well characterized and very well understood. With CS acquisition and reconstruction, the fidelity of the reconstruction to the true image can be an important unknown without prior knowledge of the true image.

Further simulation results were also conducted which we do not have space to show here. However, use of 2D encoding schemes (i.e. spiral or radial *k*-space trajectories) greatly improves reconstruction quality compared to 1D encoding schemes. This advantage is even more pronounced if the direction in which NonF encoding is applied is also rotated for different acquisition steps. Compressibility can also become more pronounced as the dimensionality of the experiment is increased, and the performance of CS improves in these regimes, as long as the compressibility basis is chosen wisely for the specific application [5].

Fig 1. Reconstruction errors in a noiseless case for different sampling schemes. Non-Fourier encoding outperforms other methods given sufficient data.



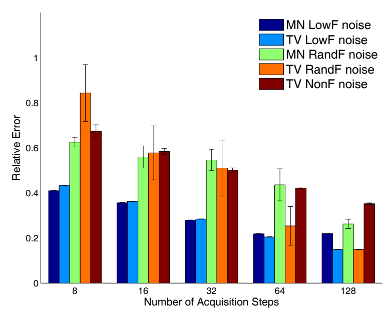


Fig 2. Reconstruction errors in a noisy case. Standard LowF acquisition provides the most consistent and most accurate results.

CONCLUSION

We have demonstrated the potential utility of non-Fourier encoding in CS MRI, which can present a large improvement over RandF sampling. We have also found that CS reconstruction can give problematic results as the SNR or the number of data points becomes more limited. In cases where reconstruction robustness and strong characterization are important, more traditional acquisition and reconstruction methods are most suitable.

REFERENCES

[1] Candès et al., IEEE Trans Info Theory 2006;52:489-509.
[2] Candès et al., Comm Pure and Applied Math 2006;59;1207-1233.
[3] Candès et al., "Near Optimal Signal" Submitted IEEE Trans Info Theory 2006.
[4] Lustig et al., ISMRM 2006; 695.
[5] Lustig et al., ISMRM 2006; 2420.
[6] Mitsouras et al., Magn Reson Med 2004;52:321-328.
[7] Rudin et al., Physica D 1994;60:677-684.