Practical Iterative MR Image Reconstruction from Very Sparse Radial Samples

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Introduction : Radially encoded MR imaging (MRI) has been well adopted in the field of fast imaging, due to its robustness to motion and relatively high signal to noise ratio (SNR) compared to the direct Fourier imaging. In many of its applications, however, the trajectory is under-sampled to reduce scan time and capture fast physiological changes. Conventional gridding reconstruction from under-sampled data introduces severe streaking artifacts and results in low SNR. Iterative MR image reconstruction algorithms based on L1 regularization and a data fidelity constraint were proposed recently [1-4]. Although the image quality was significantly improved, the computational cost for such an iterative approach was prohibitively high, especially in the 3D case. This high computational cost is mainly due to slow convergence in the iterative process. Additionally, the need to perform non-uniform Fourier transforms in each iteration hampers fast operations. In this work, we tackle these two aspects to significantly reduce the reconstruction time while achieving image quality comparable to that in [3].

Methods: The main idea behind the iterative approach [1-4] is to minimize the L1 norm of the sparse representations of the image subject to the *k*-space data fidelity. Some encouraging results with significant image quality improvement were reported in [1-4]. In particular, Chang et al. [3] incorporated Bregman iterations into the optimization and demonstrated further resolution refinement. However, the convergence in conjugate gradient (CG) algorithm used for solving the minimization problem is very slow due to the non-orthogonality of the Fourier projections. Also, during each CG iteration, the Fourier operator and its adjoint are required to be applied, respectively, to the image and the *k*-space data. A gridding step is typically needed due to non-Cartesian sampling. In this work to accelerate convergence, we generalize our previous formulation to include a *k*-space weighting **W** for the data fidelity term. For the k^{th} Bregman iteration, we have

$$\tilde{f}_{k+1} = \underset{\forall f}{\arg\min}\, \Phi(f) = \underset{\forall f}{\arg\min} \left\|\varphi(f)\right\|_{L1} + \lambda \left\|\mathbf{W}\left(\mathbf{A}f - y - v_k\right)\right\|_{L2}^2 \text{ where } v_k = y + v_{k-1} - \mathbf{A}\tilde{f}_k \text{ and } v_0 = 0. \quad (1)$$

In Eq. (1), f is a trial image; $\varphi(f)$ transforms f into a sparse representation; λ is the Lagrange multiplier; y is the measured *k*-space data vector; and matrix **A** is a Fourier operator that maps the image into *k*-space. The data fidelity term is updated in each Bregman iteration to include the *k*-space residual from the previous iteration. The optimization is solved by nonlinear conjugate gradient (CG) method. Note that with an identity weighting matrix, Eq. (1) reduces to our previous model [3]. In this work, the weighting matrix is derived for near optimal convergence of the data fidelity term, since it is usually the dominant part of the cost functional. With the solution $\mathbf{D}=\mathbf{W}^H\mathbf{W}=(\mathbf{A}\mathbf{A}^H)^{-1}$, the data fidelity is expected to converge after the first CG iteration and given an initial image f_k^0:

$$\left\|\mathbf{W}\left(\mathbf{A}f_k^1 - y - v_k\right)\right\|_{L2}^2 = \left\|\mathbf{W}\left(\mathbf{A}\left[f_k^0 - \mathbf{A}^H\mathbf{D}\mathbf{A}f_k^0 + \mathbf{A}^H\mathbf{D}\left(y+v_k\right)\right] - y - v_k\right)\right\|_{L2}^2 = \left\|\mathbf{W}\left(\mathbf{A}\mathbf{A}^H\mathbf{D} - I\right)\left(\mathbf{A}f_k^0 - y - v_k\right)\right\|_{L2}^2 = 0 \quad , \quad (2)$$

where we have used the update equation for f_k^1 in the first equality and rearranged to obtain the second equality. Consequently, the use of this weighting results in a near optimal convergence and reduces a significant number of iterations. The weighting can be alternatively viewed as an orthogonalization of the non-orthogonal Fourier operator **A**. Further investigation found that **D** is well approximated by a diagonal matrix and the density compensation function used in conventional gridding is a good choice for such a weighting. Furthermore, to improve computation speed in each iteration, we note that $\mathbf{A}^H\mathbf{D}\mathbf{A}$, which is used in the update equation and cost functional, is a block-Toeplitz matrix [5] and can be pre-calculated for each different sampling trajectory. As a result, during each iteration, only regular FFT is performed and interpolation of non-Cartesian samples, typically used in the conventional implementation, is not required.

Results: The accelerations described above were implemented to reconstruct a 2D radially-encoded phantoms acquired on a Siemens Magnetom Avanto 1.5T scanner with a TrueFISP sequence. Total variation was used in our implementation as the L1 norm term for its simplicity and robustness. The data set consists of 63 radial lines with 512 samples each, including twofold oversampling (15.75% of Nyquist sampling rate) and with 3 surface coils. The image of each channel was reconstructed using the proposed scheme independently, and the final image was obtained via root mean square of these individual images. Fig. 1 compares the reconstructed images using the conventional gridding method and the optimization model with and without a density compensation weighting. Figs. (b) and (d) both use 15 iterations (3 Bregman iterations with 5 CG each), while Fig. (c) uses 500 (5 Bregman iterations with 100 CG each). Figs. (c) and (d) have very close visual quality but Fig. (b) is very blurry suggesting that it is still very early in the iterative process. In fact, Fig. (d) shows better details than those in Fig. (c) as evidenced by the smallest black dots.

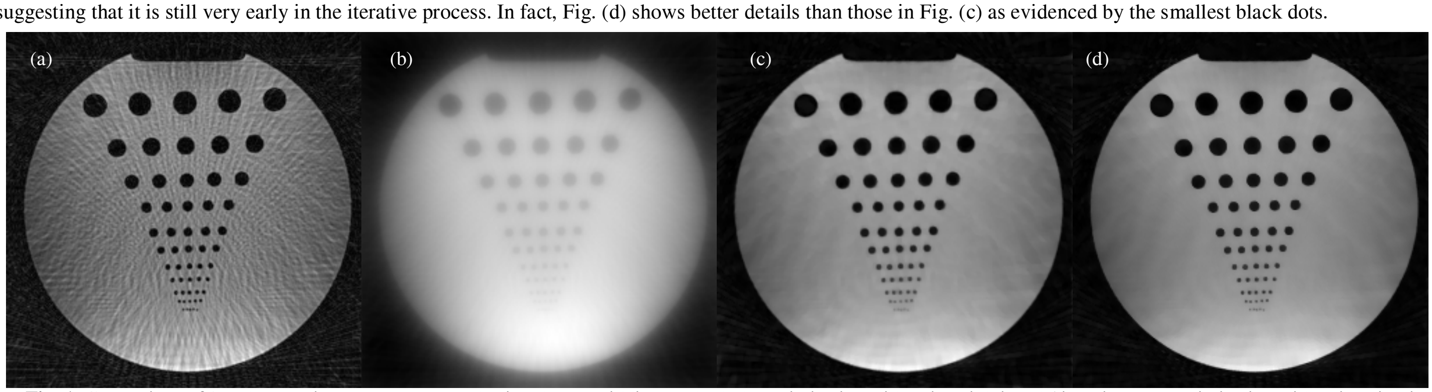

Fig. 1: Comparison of 2D phantom images reconstructed using the (a) gridding method, (b) optimization with unit weighting, 15 iterations, (c) optimization with unit weighting, 500 iterations, and (d) optimization with density weighting, 15 iterations.

Discussion and Conclusion: Two major accelerations have been developed on the L1 norm regularized optimization model for MR image reconstruction from severely under-sampled radial data. The data fidelity term was generalized to incorporate a *k*-space weighting to significantly improve the convergence rate. A fast calculation of the cost energy and its gradient was realized with a block-Toeplitz implementation. With significantly shorter reconstruction time, the weighted optimization model obtained image quality comparable to that obtained with unit weighting. Total variation worked very well here because the phantom images were piecewise constant. Other sparse representations such as a wavelet transform may also be explored for real medical images. The Lagrange multiplier leveraged the penalties between the sparsity of the transform coefficients and the data fidelity. Automatic determination of this multiplier is a direction for future research. Our proposed methods accelerated 2D iterative image reconstruction by a factor of 10-15, and will be fundamentally important for 3D applications and/or dynamic imaging.

References

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[*] This work was performed while Jiayu Song was an intern at Siemens Corporate Research, Inc.