Geographically Weighted GRAPPA Reconstruction and Its Evaluation with Perceptual Difference Model (Case-PDM)

J. Miao[1], D. Huo[2], and D. Wilson[1,3]

[1]Biomedical Engineering, Case Western Reserve University, Cleveland, Ohio, United States, [2]Keller Center for Imaging Renovation, Barrow Neurological Institute, Phoenix, Arizona, United States, [3]Radiology, Case Western Reserve University, Cleveland, Ohio, United States

INTRODUCTION: A parallel imaging technique, GRAPPA (GeneRalized Auto-calibrating Partially Parallel Acquisitions), has been used to improve temporal and/or spatial resolution [1]. Coil calibration in GRAPPA is done in fully sampled reference k-space using a least-squares technique to solve the over-determined equations. In this paper, we describe a general weighted GRAPPA method (WGRAPPA) whereby Geographically Weighted Regression (GWR) [2] is applied in the coil calibration process. Using a perceptual difference model (Case-PDM) to quantitatively, and objectively, evaluate image quality, we optimized parameters of the new algorithm and evaluated reconstructions using 4 to 32 coils and over 1,000 test images. WGRAPPA is compared to standard GRAPPA with both Case-PDM and visual inspection.

THEORY: For a set of n observations $\{x_{ij}\}$ with spatial coordinates $\{(\mu_i,\nu_i)\}$, i = 1, 2, …, n, on p independent predictor variables, j = 1, 2, …, p, and a set of n observations on a dependent or response variable $\{y_i\}$. The underlying model for GWR is

$$y_i = \beta_0(\mu_i,\nu_i) + \sum_{j=1}^{p} X_{ij}\beta_j(\mu_i,\nu_i) + \varepsilon_i \quad (1)$$

where $\{\beta_0(\mu_i,\nu_i), \beta_1(\mu_i,\nu_i), ..., \beta_p(\mu_i,\nu_i),\}$ are p+1 continuous functions of the location (μ_i,ν_i) in the study area. The ε_i is the random error term with a normal distribution $N(0,\sigma^2 I)$ .The estimator of β_i is given at each location i by a weighted least-squares approach:

$$\hat{\beta}_i = [X^T W_i(\mu_i,\nu_i) X]^{-1} X^T W_i(\mu_i,\nu_i) y \quad (2)$$

where $W_i(\mu_i,\nu_i)$ is an n by n matrix:

$$W_i(\mu_i,\nu_i) = diag(w_{i1}, w_{i2,} w_{i3}, ..., w_{in}) \quad (3)$$

In equally weighted regression models, the values of $W_i(\mu_i,\nu_i)$ are constant. In the GWR model, $W_i(\mu_i,\nu_i)$ varies with the location i depending on the distance between i and its neighboring locations. The above process is repeated for each observation in the data, and consequently, a set of parameter estimates is obtained for each location. Here we give one example of GWR in the GRAPPA coil calibration process, where we weight the auto-calibration signal (ACS) samples in a VD acquisition scheme. The spatial weighting function used in this example is given below where wij increases with increasing distances from the k-space center because of the variation of k-space signal which is described in [3].

$$w_{ij} = 1 - \exp(-(d_x - x_{shift})^2 / 2\sigma_x^{\,2} - (d_y - y_{shift})^2 / 2\sigma_y^{\,2}) \quad (4)$$

Above the d's are distances from each data point to the center of k-space in each coil, x_{shift} and y_{shift} are the shift in frequency encoding direction and shift in phase encoding direction in pixels from the center of k-space, σ is the width of the Gaussian function. WGRAPPA uses the same process of reconstructing missed signals as the conventional GRAPPA does.

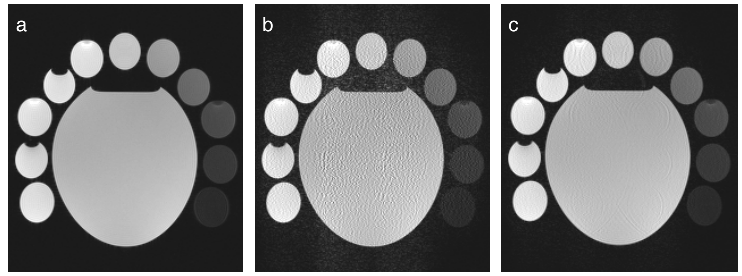

Figure 1. 4-channel phantom images of (a) reference, (b) GRAPPA reconstruction, (c) WGRAPPA reconstruction. All reconstructions were done with ORF's of 4 and ACS's of 80 (total R=1.9).

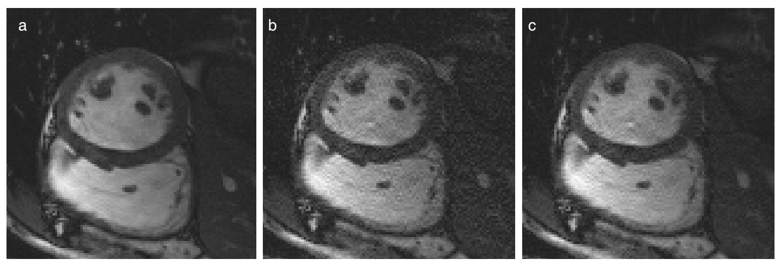

Figure 2. 8-channel cardiac images of (a) reference, (b) GRAPPA reconstruction and (c) WGRAPPA reconstruction. All reconstructions were done with ORF's of 4 and ACS's of 40 (total R=2.5).

METHODS: In this experiment, five different image datasets with different content (phantom, cardiac, liver and brain) were used. Phantom image datasets were fully acquired using Turbo SE sequence with 4, 8, and 12 channel head coils and a 32 channel cardiac phased coil. In vivo image datasets were acquired from healthy volunteers. For each dataset, full-sampled data were acquired and used to reconstruct the reference image. Over 1000 test images from a variety of conditions (5 in vivo and phantom image datasets, 5 coil conditions, 3 acceleration factors [ORF's], 3 ACS's, 2 ACS integration conditions, 10 width conditions and 10 shift conditions) using both GRAPPA and WGRAPPA. We used CASE-PDM to obtain a single set of optimized parameters in Eq. 4.

RESULTS: We found that $x_{shift} = y_{shift} = 0$ and $\sigma_x = 80, \sigma_y = 30$ gave the lowest PDM score, and visually gave good suppression of noise and artifact. Figure 1 shows the reconstruction results of 4-channel phantom image from GRAPPA and WGRAPPA with Outer Reduction Factor (ORF) of 4 and ACS integration in final reconstruction. With the proposed method for coil calibration, WGRAPPA can improve the image quality and gives an almost perfect image. Figure 2 shows the reconstruction results of 8-channel in vivo cardiac image from GRAPPA and WGRAPPA with ORF's of 4 and ACS integration. Noise and aliasing artifacts are suppressed with the proposed method as compared to standard GRAPPA. Even with only one filter, WGRAPPA gives better reconstructions of k-space having lower k-space residuals than GRAPPA. With the optimized parameter set, WGRAPPA gave a better PDM score than GRAPPA, for every reconstructed test image.

DISCUSSION: The WGRAPPA algorithm improves fitting accuracy and reconstructed image quality. WGRAPPA is robust. We tested only one filter function (Eq. 4), but many other functions are possible. Yet another extension is the use of multiple filter functions, which we have found to be quite advantageous, especially in the case of a pre-scan where ACS lines cannot be integrated into the final reconstruction (result not shown). The method can be extended to dynamic imaging (tGRAPPA). In general, WGRAPPA with appropriate weighting functions and locations includes all GRAPPA k-space weighting methods, including the variable density sampling method [3]. With one weighting function, reconstruction quality was limited in the case of a VD acquisition without ACS integration. There were obvious low frequency artifacts even though noise was greatly suppressed. This limitation is reduced when a more complicated weighting scheme is introduced (not shown). We believe that Case-PDM is suitable and advantageous in MR algorithm evaluation and parameter optimization.

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