Time-Segmented Spin Domain Method for Fast Large-Tip-Angle RF Pulse Design in Parallel Excitation

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INTRODUCTION: Most methods for RF pulse design in parallel excitation use the linear small-tip-angle approximation. However, pulses designed using these methods produce distorted excitation at large tip angles (1). Bloch equation-based pulse design methods may be extended to this problem (2,3), but they require substantial computation to evaluate the excitation produced by an RF pulse, as well as derivatives of the pulse design cost function. Here we introduce a method that replaces the exact Bloch equation with a fast model, using a time segmentation scheme to divide the excitation period into time segments and approximate the Bloch equation solution within each segment using the small-excitation approximation (4). We demonstrate that this method provides a rapid means for designing large-tip RF pulses in parallel excitation, e.g., saturation and inversion pulses.

METHODS: The well-known Shinnar-Le-Roux pulse design algorithm (5) is formulated in the spin domain. In this domain, the rotation produced by an RF pulse may be exactly represented by the successive multiplication of a series of rotation matrices, each of which represents the rotation produced by one sample of the pulse:

$$\begin{pmatrix}\alpha_{net}\\ -\beta^*_{net}\end{pmatrix} = \begin{pmatrix}\alpha_{N_t} & \beta_{N_t}\\ -\beta^*_{N_t} & \alpha^*_{N_t}\end{pmatrix}\cdots\begin{pmatrix}\alpha_1 & \beta_1\\ -\beta^*_1 & \alpha^*_1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}. \qquad [1]$$

For a given pulse, computing $(\alpha_{net},\beta_{net})^T$ and its derivatives with respect to RF samples is computationally expensive, particularly for multi-dimensional parallel excitation pulse design. To reduce computation, we group contiguous RF time points together into larger time segments, and use the small-excitation approximation to predict the rotation parameters (α_i,β_i) for each segment i as:

$$\alpha_i^*(\mathbf{x}) = e^{\frac{i}{2}\mathbf{x}\cdot\mathbf{k}_i(0)}, \quad \beta_i(\mathbf{x};b) = \frac{i\gamma}{2}e^{\frac{i}{2}\mathbf{x}\cdot\mathbf{k}_i(0)}\sum_{r=1}^{R} s_r^*(\mathbf{x})\int_{t_i}^{t_{i+1}} b^*_{1,r}(t)e^{-i\mathbf{x}\cdot\mathbf{k}_i(t-t_i)}dt, \qquad [2]$$

where $s_r(\mathbf{x})$ is coil r's transmit sensitivity, $b_{1,r}(t)$ is coil r's RF pulse, and $\mathbf{k}_i(t)$ is segment i's excitation k-space trajectory (4). Eq. 2 is analogous to the small-tip approximation (6), and may be discretized in a manner analogous to the small-tip parallel excitation pulse design technique of (7) and computed efficiently using non-uniform FFT's (8). The rotation parameters from each segment are then used to compute the net rotation produced by the entire pulse via Eq. 1, where each matrix now represents the rotation produced by many contiguous pulse samples. We then form a least-squares cost function in terms of the final magnetization state after applying the pulse. Assuming magnetization is initially at equilibrium, this is given by:

$$m_{xy} = -2\alpha^*_{net}\beta^*_{net}m_0, \quad m_z = \left(\alpha_{net}\alpha^*_{net} - \beta_{net}\beta^*_{net}\right)m_0. \qquad [3]$$

We can derive the gradient of our cost function with respect to the RF samples, and minimize it using non-quadratic conjugate gradient (9). Segment placement is chosen based on initial small-tip designed seed pulses, by accruing RF time points into a segment until a threshold integrated power level is reached somewhere in the object.

We verified our method in simulation. Transmit sensitivity patterns were approximated by the receive patterns of an 8-channel receive coil array. An ROI was defined by thresholding an image of the phantom, and the desired pattern was defined as a square of 180° flip angle, situated off-center in the ROI (figure 1). The applied excitation trajectory was an undersampled spiral with excitation FOV of 7cm, and spatial resolution 20/32cm, corresponding to pulse length 2.4ms and 595 RF pulse samples per coil. The desired excitation pattern for pulse design was specified on a 64x64 grid with 20cm FOV, and final error was calculated on a 128x128 grid with 20cm FOV. Normalized RMS error (NRMSE) was calculated between the desired and excited flip angle patterns, and average flip angle was computed within the square. 70 time segments and 30 CG iterations were used.

RESULTS: Flip angle patterns excited by small-tip designed pulses and time-segmented spinor designed pulses are shown in Fig. 2a and b, respectively. The small-tip designed pulses represent those that produce the lowest excitation error with respect to Tikhonov regularization parameter (6), and it can be seen in Fig 2a that the excitation pattern is heavily distorted, with NRMSE=0.13, average flip angle 162°, and standard deviation 7°. In contrast, pulses designed using our technique produce undistorted excitation (2b) with NRMSE=0.06, average flip angle 171°, and standard deviation 1.5°. Compute time for our technique was 17min on a 3Ghz P4 PC in MATLAB, compared to 3s for small-tip pulse design. In summary, the proposed approach provides rapid design of large-tip RF pulses for parallel excitation, e.g., B1+-compensated inversion pulses.

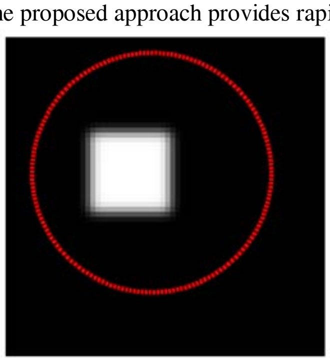

Figure 1: Desired excitation pattern (5x5cm square) and ROI (circle) for pulse design and error calculation.

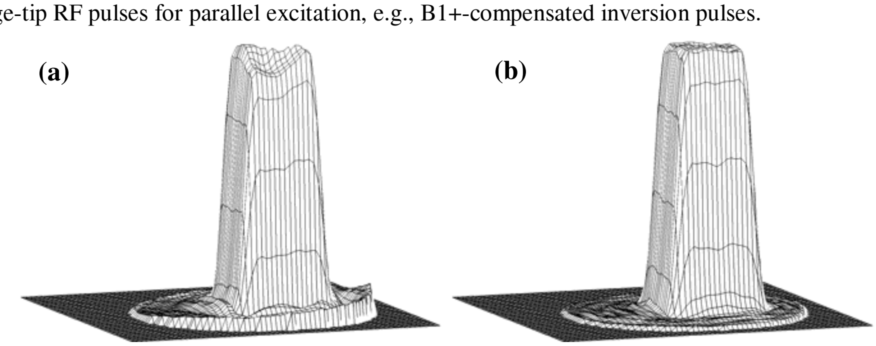

Figure 2: Simulated flip angle patterns. **a:** Small-tip designed pulses, NRMSE=0.13, average flip angle=162°. **b:** Time-segmented spinor designed pulses, NRMSE=0.06, average flip angle=171°.

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