Iterative GRAPPA: a General Solution for the GRAPPA Reconstruction from Arbitrary k-Space Sampling

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Introduction: Recently, several non-Cartesian solutions for GRAPPA [1] reconstruction have been proposed [2-4]. These methods are either approximate, or tailored for specific sampling trajectories. Inspired by the generality of non-Cartesian SENSE [5], we propose a general accurate solution for GRAPPA reconstruction from arbitrary k-space sampling. The reconstruction is formulated as an optimization, forcing consistency with the calibration and acquisition data. The optimization has a simple, efficient and rapidly converging iterative solution. We demonstrate a reconstruction from randomly under-sampled k-space.

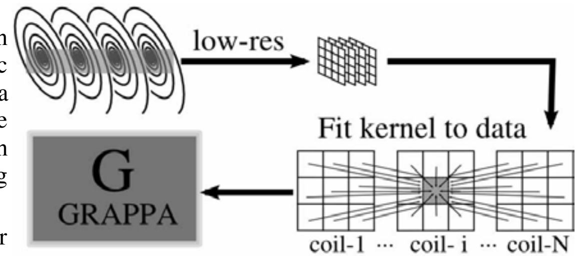


***Figure 1**: Calibration. Unlike traditional GRAPPA, the kernel entries are fully occupied and do not assume any specific under sampling.*

Theory: The GRAPPA algorithm synthesizes missing k-space data points using a linear combination of multi-coil neighboring points. The linear combination coefficients, i.e, the GRAPPA operator, are obtained by calibration on a fully sampled k-space subset.

Here, we examine the case of arbitrary sampling of k-space. We first assume that there is always a calibration region from which we can generate a Cartesian calibration area (for example: by regridding) and obtain a Cartesian GRAPPA operator as depicted in Fig. 1.

Ideally, if we were able to reconstruct the coil images correctly (by any possible method) and get a full Cartesian k-space image, applying our GRAPPA operator on the reconstructed k-space (synthesized+acquired) should yield the exact same data – because it should be consistent with the calibration data. This observation leads to a new formulation for solving the GRAPPA reconstruction. We search for a Cartesian k-space image that is consistent (up to $\varepsilon<noise$) with the acquired data from the scanner (not necessarily Cartesian), and for which applying the GRAPPA operator results in the minimum residual – i.e., also consistent with the calibration data. This desired k-space image is the solution for the following optimization:

$$\text{minimize } ||Gx - x||^2 \text{ subject to: } || Dx - y ||^2 < \varepsilon. \qquad (1)$$

Here, $x=[x_1,x_2,\ldots,x_n]^T$ is the desired full-grid multi-coil reconstructed k-space image, $y=[y_1,y_2,\ldots,y_n]^T$ is the acquired multi-coil data from the scanner, G is our <u>Cartesian</u> (!) GRAPPA convolution operator, and D is a resampling operator. In Cartesian grid sampling, D selects the acquired k-space lines from the full grid, in non-Cartesian sampling D is a regridding convolution operator. Eq.1 can be solved in many ways; a simple solution is to solve the non-constrained version of the optimization, i.e., minimize $|| Dx - y ||^2 + \lambda(\varepsilon)||Gx - x||^2$ iteratively using Conjugate Gradients (CG) methods (see Fig.2). The operators G* and D* are conjugates of G and D and are Cartesian convolutions as well.

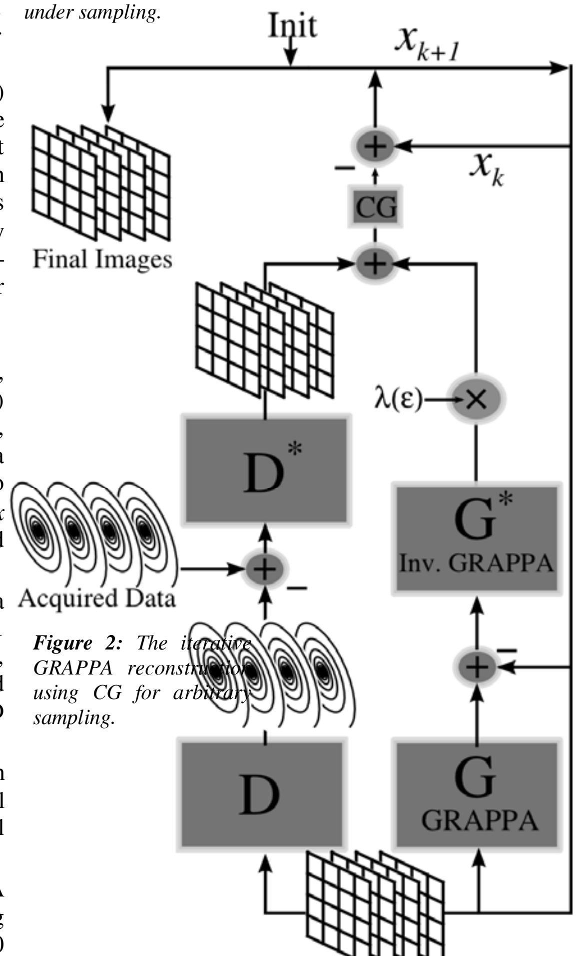


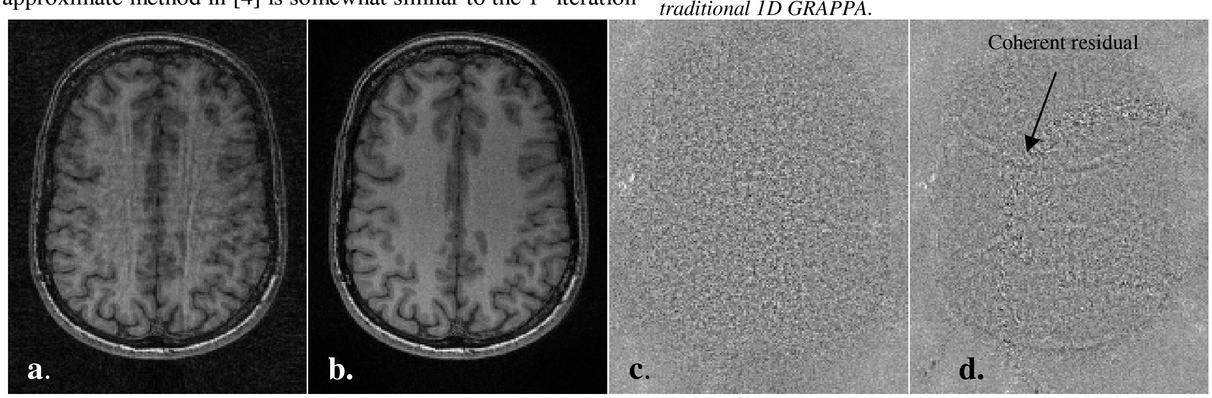
***Figure 2:** The iterative GRAPPA reconstruction using CG for arbitrary sampling.*

Methods: To demonstrate the generality of the approach we tested the reconstruction on a randomly under-sampled data set by under-sampling the phase encodes (2-fold) of a T1 weighted, 3D SPGR sequence of a brain (256x180x160, res=1 mm, TR/TE=32/5 ms, flip=20). The data was acquired on a 1.5T GE Signa Excite scanner using an 8-channel head coil. Five CG iterations were performed. The result was also compared to a traditional 1D GRAPPA reconstruction.

Results: The iterative method exhibits good reconstruction from highly non-uniform sampling (Fig 2a-b.). The error image shows only noise (Fig. 2c) whereas traditional GRAPPA reconstruction from uniform under sampling (Fig. 2d) exhibits a typical residual coherent. This shows the attractiveness of non-uniform GRAPPA.

Discussion and Conclusions: We presented the formulation and solution for the GRAPPA reconstruction from arbitrary k-space sampling. The reconstruction is efficient, involving only Cartesian convolution operations, and normally converges within less than 10 iterations. This approach opens the attractive possibility of a simple and accurate solution for non-uniform and non-Cartesian parallel imaging. Also, in this framework it is also natural to apply image-based regularizations currently used in SENSE reconstruction. It is interesting to note that the approximate method in [4] is somewhat similar to the 1st iteration in the proposed method.

***Figure 3:** (a) typical artifacts of random-under sampling. (b) the iterative recon. (c) random under sampling iterative GRAPPA error image (windowed x7) compared to a full acquisition. (d) typical error image (windowed x7) of traditional 1D GRAPPA.*



References: [1] Griswold et al., MRM 47:1202-10 (2002) [2] Heidemann et al. MRM 56(2):317-26 (2006) [3] Heberlein et al 55(3):619-25 (2006) [4] Hu et al, ISMRM 2006:10 [5] Pruessmann et al., MRM 46:638-51(2001)