Reciprocity and Gyrotropism in Magnetic Resonance Transduction

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Introduction and Theory: Harrington and Villeneuve (*1*) have formulated a theory of radio reception in gyrotropic media, i.e. media in which the AC susceptibility tensor is transposed upon reversal of an external DC field. The applicability of this work to the problem of magnetic resonance transduction have been noted (*2,3*), and a key result given for the receiver emf:

$$i\omega\mu_0 \int \hat{\mathbf{H}}_c \boldsymbol{\chi} \mathbf{H}_c dV = \int \mathbf{E}_s \bullet \hat{\mathbf{J}}_c dV = I_c \int \mathbf{E}_s \bullet d\hat{\mathbf{s}} \quad [1]$$

Here the subscript *c* (for coil) indicates the RF fields $\mathbf{H}_c$ arising from impressed RF current $\mathbf{J}_c$ in an RF coil, and subscript *s* (for sample) indicates induced fields in a sample irradiated by that coil. Also, the caret signifies fields and currents obtaining when when the DC polarizing field is reversed; further, the volume integral of $\mathbf{E}\cdot\mathbf{J}$ is converted to line integral over the periphery of the coil by assuming filamentary currents. The susceptibility tensor for NMR, $\boldsymbol{\chi}$, (*3,4*) is appended below for reference; its asymmetry embodies the gyrotropism due to nuclear spins; for small tip angles $f(\omega)$ is a complex Lorentzian (*4*). For two identical, linearly polarized RF coils, operating in tandem as a transmit-receive pair, the carets are not needed, and matrix multiplication on the left of Eq. [1] give the scalar : $\mathbf{H}^{(r)}\boldsymbol{\chi}\mathbf{H}^{(t)} = f(\omega)(H_x^{(r)} - iH_y^{(r)})(H_x^{(t)} + iH_y^{(t)})$ where we have labelled the field components of the transmit and receive coils with superscripts *t* and *r* . Putting this into Eq. [1] gives the received voltage in terms of the *antenna patterns* $(H_x \pm iH_y)$ for identical coils, the RX (receive) pattern with negative H_y, the TX (transmit) with positive.

Results and Discussion: Figure 1 shows a dual surface array, and Fig. 2 gives phantom images and simulations from its individual coils operating as receivers, with an external body transmitter. The divot of lost intensity (left) arises from reactive crosstalk (*5*), and is of key importance, causing the two RX patterns to differ, despite symmetry. Field components of the coils are mixed by the crosstalk, but weakly enough for each coil to maintain its primary identity. For uniform excitation, the complex RX patterns – including crosstalk, but, for convenience, denoted by $(H_x^{(1)} - iH_y^{(1)})$ and $(H_x^{(2)} - iH_y^{(2)})$ -- give the complex images directly; the indices 1 and 2 label left and right coils. Per Harrington and Villeneuve, the RX patterns (and divot artifact) must exchange sides if B_0 is reversed, as illustrated in Fig 3a - d. But also, the *RX patterns with field reversed* are --in gyrotropic media-- the *TX patterns for field unchanged* . Therefore an experiment using the array as a TRX device, with TX power applied to one port or the other, should give distinct results, as simulated in Figs. 3e and 3g and confirmed in experimental images 3f and 3h. Note the characteristic umbrella pattern predicted in 3e and observed in 3f, and the cat's eye predicted in 3g and observed in 3h. The simulations are per the product $\mathbf{H}^{(r)}\boldsymbol{\chi}\mathbf{H}^{(t)}$, (see above), and the agreement of predicted and observed patterns (umbrella to umbrella and cat's eye to cat's eye) tends to confirm the theory of gyrotropic reciprocity in NMR.

Fig. 1: Dual array for proton imaging at 3.0 T; note vitamin cap fiducials (yellow lozenges).

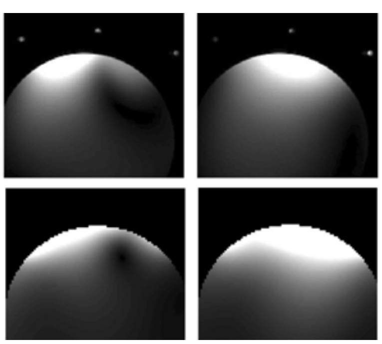

Fig 2: Above: images from coils 1 & 2 (l & r) of dual array; sphere, 9.25 cm rad., 25 mM aqueous NaCl; below simulations: magnitudes of RX patterns $(H_x^{(1)} - iH_y^{(1)})$ and $(H_x^{(2)} - iH_y^{(2)})$.

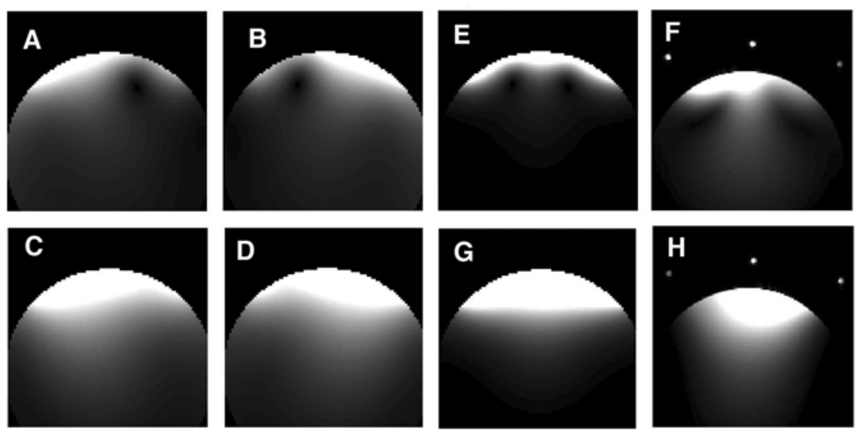

Fig. 3a & d: simulate RX patterns of coils 1 and 2 (cf Fig. 2); 3b and 3c, same with B_0 reversed, i.e. with H_y positive. 3e is product of 3a & b; 3g product of 3c & d; 3f is the image of dual array in TRX mode, TX applied to right coil. 3h, same, but TX left coil; note fiducials (cf Figs. 1 & 2)

References: 1. R. F. Harrington & A.T. Villeneuve, IRE, **MTT6**, 308 (1958).
2. J. Tropp, *Physical Review A* in press (2006).
3. J. Tropp, Proc. 11th ISMRM, 2552, (2003).
4. T. Ibrahim, *Magn. Reson. Med.*, **54**, 667, (2005)
5. C. P. Slichter, *Principles of Magnetic Resonance,* Harper & Row, 1963, Ch 2.
6. J. Tropp & T. Schirmer, *J. Magn. Reson*. **151**, 146 (2001)

$$\boldsymbol{\chi} = f(\omega)\begin{bmatrix} 1 & i & 0 \\ -i & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$