Image entropy of undersampled data: a useful measure of aliasing severity

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Introduction Undersampling of dynamic data may be used to make acquisition more efficient by forcing spatially distinct voxels to share their temporal bandwidth; this can be a benefit if only a small number of voxels require the full sampling bandwidth. Data are acquired in the k-t domain (k is spatial frequency and can be multidimensional, t is time) but are usually analysed in the conjugate x-f domain (x is space, f is temporal frequency). The intermediate spaces k-f and x-t, produced by Fourier transform in only one dimension from k-t are also of interest. Reconstruction from regularly undersampled multi-coil data (with k-t SENSE, x-f choice and others) can be accurately achieved in x-f space provided the number of signals that significantly contribute to an aliased voxel is small enough to be separated by a parallel imaging reconstruction (1). If this condition is violated it is necessary to use filtering or other methods that can at best approximate the correct solution. This implies that degree of signal overlap in x-f space is a key indicator that an undersampled dataset can be reconstructed accurately. However, to use this as a metric requires detailed analysis of the dynamic properties of the data, so a more generic indicator would be preferred. A conceptually appealing candidate is image entropy (H) (2), and here we consider its use as an indicator of signal overlap caused by aliasing.

Method and results To explore the relationship between H and signal overlap, fully sampled k-space data for a time series of 2D images was undersampled with various different undersample patterns, all for the same undersample factor Q in dimensions (k_y-k_z-t) – equivalent to undersampling a 3D Cartesian scan. Patterns were constructed by undersampling k_y by factor Q before cyclically shifting this in k_z and t. The shift for each point in (k_z,t) may be calculated as $\{k_z \times dz + dt \times (t\%Q)\}\%Q$ where % signifies the modulo operation, and dz and dt are parameters with ranges [1,Q] and [1,Q-1] respectively. Each (dz,dt) pair distributes aliases differently in x-f space. Results displayed in figure 1 were calculated from fully sampled short axis cardiac data with Q=5, giving 20 possible patterns. H was calculated for each pattern, as was a crude measure of the quantity of overlap of 'dynamic' signal. This was made by using a threshold on the fully sampled data in x-f space to give a binary map of 'dynamic information' so that for each alias pattern, the percentage of dynamic information aliasing onto other dynamic information could be estimated. The results from these methods (fig 1) were found to be inversely correlated with a coefficient of -0.86. There were no strong correlations between overlap area and H in the x-t, k-t or k-f domains. An explanation is that there are many static voxels in the image, meaning that large amounts of 'area' in x-f space contain no information. Aliasing dynamic information onto this area does not create signal overlap, and so appears to increase the information content. Conversely by aliasing dynamic onto dynamic, the total amount of apparent information does not increase. Following from this result we take H to be a surrogate measure of the amount of signal overlap in aliased data with a higher H signifying less overlap

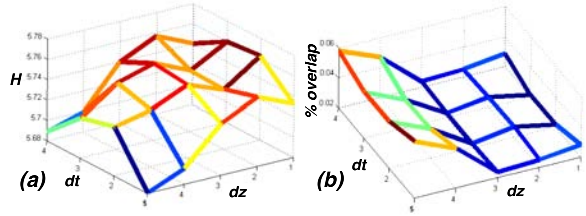

Fig 1 **a)** Entropy in x-f space for each undersample pattern. **b)** Area overlap in x-f space. The two correlate with R=-0.86.

Further consider variation of H as a function of Q for each space, x-t, x-f, k-t, k-f. Note that the absolute value of H for a given data set is different in the different spaces and both their absolute and relative sizes depend on image content. For this reason we have normalised H to 1 for fully sampled data in figure 2. It is instructive to consider a model situation where there is a single small dynamically changing object in an otherwise empty field of view (FOV). Since there is nothing in the background and the dynamic object was very small (occupying only 0.5% of the FOV by area), aliasing simply replicates information onto empty space and hence H(x-t) and H(x-f) both increase with Q (fig 2a). As expected H(k-t) falls as Q rises since increasing numbers of samples are set to zero, however we might have expected H(k-f) to increase; the fact that it does not demonstrates information loss from aliasing. This explains why x-f is the better domain in which to analyze and reconstruct. Adding a static background image to this model changes matters; H(x-t) does not increase as quickly as H(x-f) any more because aliasing of static regions onto other static regions causes beating and flickering in the x-t domain, while in the x-f domain information separation is preserved. A model with only static features helps to explain this (fig 2c); interestingly for this model, H(k-f) grows with Q because there is no signal overlap as all k values contain only static information.

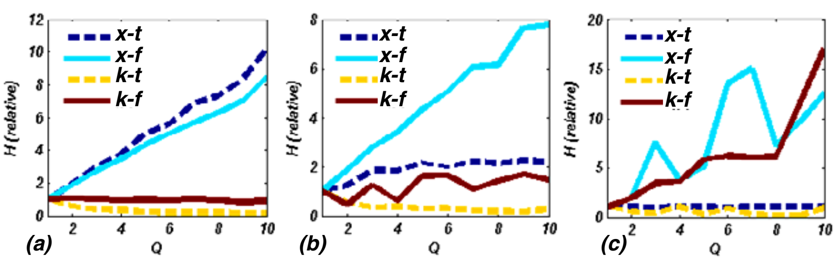

Fig 2 H vs α for models **a)** with dynamic information and no background **b)** with both and **c)** with only static background information. H is normalised to H for Q=1. Single dynamic region occupied approx 0.5% of the FOV.

This analysis shows that in general H(x-f) increases as a function of Q when there is only a small dynamic feature, and that the addition of background information complicates matters but does not stop this trend. Figure 3 demonstrates H as a function of size of dynamic content: a circle of rapidly changing dynamic information was placed in the centre of a blank background, the plot shows H for Q=4 as the radius of the circle is increased. As the object size increases, the empty regions of x-t and x-f decline so the apparent amounts of information H(x-f) and H(x-t) initially increase, but then begin to saturate as the radius passes ¼ of the FOV since then increases in radius lead to overlap as well as occupation of empty space and the total information rises more slowly. In contrast H(k-t) and H(k-f) both show a decreasing trend with no initial increase.

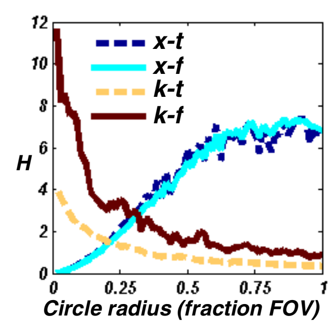

Fig 3 Circle containing dynamic information on blank background. H vs radius for Q=4.

Conclusions Image entropy in the x-f domain (H(x-f)) can be a good relative indicator of the severity of alias overlap. The strength of the technique lies in making comparisons rather than absolute judgements. For example given a pilot scan or motion model, it is possible to use H(x-f) to determine the best undersample pattern to use. At higher undersample factors H(x-f) begins to saturate, indicating that capacity to carry new information has been exhausted – this could provide a signature to indicate the maximum acceleration factor that is recoverable for a given dynamic object. X-f is the only one of the four spaces for which H systematically initially increases with aliasing regardless of image content, provided the dynamic components only occupy part of the FOV. This supports the intuitive view that x-f is the appropriate domain for reconstructing regularly undersampled data.

References 1) Malik SJ et al., MRM. 2006 Oct;56(4):811-23 2) Shannon CE, *The Bell System Technical Journal* Jul-Oct 1948

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