Radial k-t FOCUSS: k-t space Focal Underdetermined System Solver using radial trajectory

J. Yoo[1], and J. Ye[1]

[1]KAIST, Daejeon, Korea, Republic of

Introduction :   Recently, many dynamic MR image reconstruction techniques   from sparse k-space samples have been investigated. The main advantage using sparse sample is the fast data acquisition which increases the temporal resolution. However, due to the limited samples, the spatial resolution is often sacrificed.   In order to improve the reconstruction quality, we exploit the fact that the periodic motions such as cardiac is sparse in x-f space, and formulate the problem as an optimization problem under sparse constraint. Another type of sparse reconstruction algorithm called the FOCal Underdetermined System Solver(FOCUSS)[1] is employed for dynamic imaging using radial trajectory. It turns out that the existing k-t BLAST corresponds the first iteration of the our FOCUSS algorithm, and our algorithm outperforms the k-t BLAST by applying more FOCUSS iterations.

Theory :   FOCUSS is an algorithm to obtain a sparse solution in an underdetermined linear inverse problem such as

$$y = Ax \tag{1}$$

Now, let us consider the following reweighted norm solution: Find $x = Wq$ $\hspace{4em}$ (2)

where x is the unknown image, W is a weighting matrix, and q is computed by solving the constrained minimization problem:

$$\min \ \|q\|_2 \ \text{, subject to } \ \| Awq - y \| < \varepsilon \tag{3}$$

then the optimal solution is given by

$$x = \Theta A^H (A\Theta A^H + \lambda I)^{-1} y, \quad \Theta = WW^H \tag{4}$$

The FOCUSS algorithm updates the weighting matrix W at each iteration using the solution from the previous iteration, and it is shown that the resultant solution is maximally sparse [1]. For the dynamic MR imaging such as cardiac, the cine image on x-f domain is sparse due to the periodic motion of heart. Therefore, our approach is to exploit the sparsity in x-f space using FOCUSS. We term this algorithm as radial k-t FOCUSS.   More specifically, the forward imaging equation in radial trajectory can be written by

$$y = Ax = M\Phi x \tag{5}$$

where y is measurement (sinogram), x is original signal in x-f domain, M is radon transform, and Φ is Fourier transform along temporal direction. In order to exploit the sparsity in x-f domain and reduce the angular sampling without incurring resolution loss, we employ the FOCUSS described in Eq. (3) to Eq. (4), and update W matrix at each iteration using the previous x-f solution until the solution converges. As shown in (4), the first iteration of the FOCUSS is identical to the k-t BLAST algorithm [2]. Hence, our algorithm encompasses the existing k-t BLAST as a special case and overcomes its artifacts using additional FOCUSS iteration.

Result : In order to evaluate the performance of radial k-t FOCUSS, we apply it to a   cardiac data. For a simulation,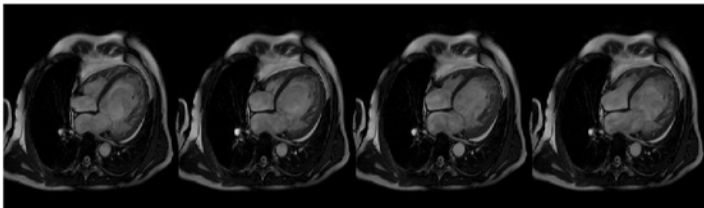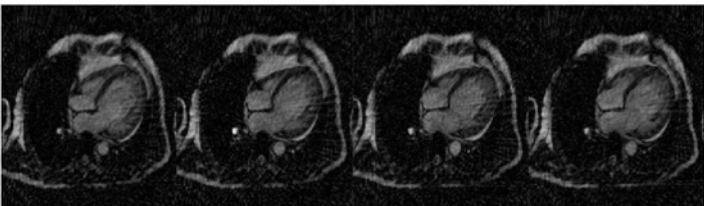 we have acquired 25 frames of ful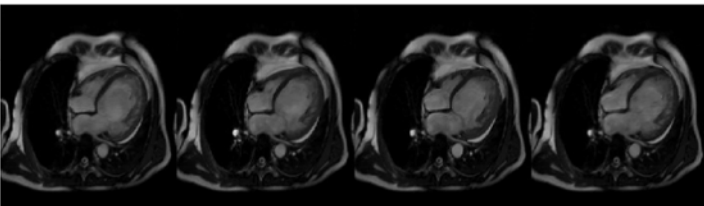l k-space cardiac sequence and simulated the undersampled radial trajectory. The full k-space data image array is 256 x 256 pixels, and the FOV was 192mm x 192mm. In this simulation, 36 radial views are obtained at each time frame which corresponds to the reduction factor of five. Fig. 1 is the original cardiac cine images, and Figs 2 is the estimation using the conventional inverse radon transform using 36 views. Angular artifacts are observed. Fig. 3 is the reconstruction using our radial k-t FOCUSS method. Aliasing artifacts from the conventional algorithm are removed using radial k-t FOCUSS.

Fig.1: original cardiac cine.

Fig.2: Inverse radon image using 36 views.

Fig.3: radial k-t FOCUSS results

Conclusion : We proposed the radial k-t FOCUSS algorithm to obtain sparse solutions from angularly undersampled k-t data. The radial k-t BLAST algorithm is a special case of our radial k-t FOCUSS. Simulation results demonstrated that the proposed radial k-t FOCUSS algorithm is an effective for dynamic MR imaging applications.

Reference

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