Self-Calibrated GRAPPA Operator Gridder (SC-GROG)

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Introduction: The GRAPPA Operator Gridder (GROG) has been recently demonstrated for the regridding of non-Cartesian datasets [1]. By using the properties of the GRAPPA Operator described in [2] and [3], a general equation for shifting points in arbitrary directions and distances in k-space can be written: $\vec{S}(k_y + m\cdot\Delta k_y, k_x + n\cdot\Delta k_x) = \hat{G}_x{}^n \cdot \hat{G}_y{}^m \cdot \vec{S}(k_y, k_x)$ or $\vec{S}_{Cart} = \hat{G}_x{}^n \cdot \hat{G}_y{}^m \cdot \vec{S}_{non-Cart}$

where n and m are the distances to be shifted in the x and y-directions, respectively, and G_x and G_y are base weights for a shift of $1\Delta k_x$ or $1\Delta k_y$. This equation allows non-Cartesian points to be shifted to the nearest Cartesian locations. However, for most trajectories, a separate Cartesian calibration dataset was needed to determine the base weights. In this abstract, a self-calibrating scheme for GROG is proposed which allows the base weights G_x and G_y to be calculated directly from the data themselves. Using the pairs of acquired non-Cartesian points that are close to each other (i.e. $n<1\Delta k_x$ and $m<1\Delta k_y$), the equation above can be either solved iteratively or analytically for G_x and G_y. For radial and spiral applications, which are quite common in non-Cartesian imaging, the GROG equations above can be solved analytically; this approach is described here.

Methods: For each projection in a multi-channel radial acquisition, the following equation can be written and solved for G_θ:

$\vec{S}(\theta, k_{r+1}) = \hat{G}_\theta \cdot \vec{S}(\theta, k_r)$ where $\hat{G}_\theta = \hat{G}_x{}^n \cdot \hat{G}_y{}^m$

Using the angular weight sets for each projection, and taking the matrix logarithm of the resulting set of equations yields the following:

$\ln(\hat{G}_{\theta 1}) = n_1 \cdot \ln(\hat{G}_x) + m_1 \cdot \ln(\hat{G}_y)$

$\ln(\hat{G}_{\theta 2}) = n_2 \cdot \ln(\hat{G}_x) + m_2 \cdot \ln(\hat{G}_y)$

$\vdots$

These equations can be reordered into another set of linear matrix equations, which can be solved coil-by-coil (i.e. a and b run from 1 to NC): $\ln(\vec{G}_\theta(a,b)) = [\vec{n}, \vec{m}] \cdot [\ln(G_x(a,b), \ln(G_y(a,b)]$

By taking the pseudo-inverse of this equation of the distance matrix, the matrix logarithms of the weights sets can be found; the base weights themselves are calculated by taking the matrix exponential of the logarithmic weights. Although this method was developed for radial trajectories, Archemidian spiral trajectories can be resorted (using time domain interpolation, if necessary) into quasi-radial trajectories, to which the same self-calibration method can be applied to arrive at the base weights.

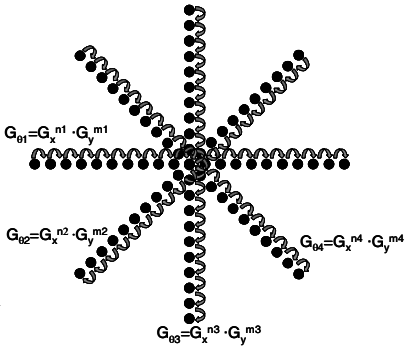

Figure 1: Calculation of self-calibrated GROG weights for the radial trajectory begins with the calculation of angular weights, G_θ, along each projection. These angular weights are simply the base weights, G_x and G_y, to the power of the distance between points in each direction.

The self-calibrating GROG method is demonstrated here for radial and spiral trajectories. In vivo radial data (12 channel, 256 projections, 512 read-out points) were acquired using a 1.5 T Siemens Avanto scanner (Siemens Medical Solutions, Erlangen, Germany), and in vivo data spiral data (8 channel, 4 spiral arms, 10472 read-out points) were acquired using a 3T Trio scanner (Siemens Medical Solutions) using a constant-linear-velocity trajectory. Before calculation of the weights, the spiral trajectories were interpolated to yield constant-angular-velocity trajectories, which can be resorted into radial trajectories. The base weights G_x and G_y were calculated using the formulas given above, and the data were gridded using Equation 1 with the appropriate n and m values. As a reference image, the radial and spiral data were also regridding using the gold-standard convolution gridding of Jackson et al [4], with a Kaiser-Bessel window (width =3), a twice oversampled grid (s=2), and the standard Ram-Lak density compensation function.

Results: The radial self-calibrated GROG and gold-standard convolution gridding images are shown in Figures 1a and 1b, and the corresponding spiral images are shown in Figures 2a and 2b. As can be seen from the images, the contrast and resolution are not affected by the use of SC-GROG.

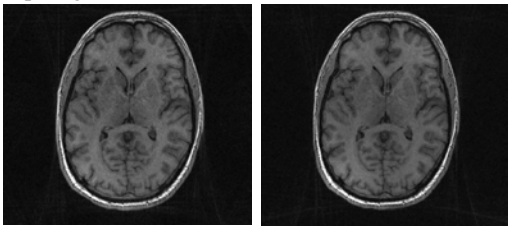

Figure 2 left: Radial image regridded with SC-GROG.
Figure 2 right: Gold-standard convolution gridding radial image.

Discussion: SC-GROG is a method by which non-Cartesian data can be resampled onto a Cartesian grid without the need for additional calibration data (as in GROG). The self-calibrating property of SC-GROG leads to better image quality than standard GROG, as there are no data mismatches between the non-Cartesian data and the Cartesian dataset. In addition, SC-GROG requires no additional information or parameters besides the data and trajectory; no DCF or convolution window must be specified, as in convolution gridding. The self-calibrating GROG method also works for 3D non-Cartesian datasets (data not shown), and requires far less memory than standard gridding methods. Finally, SC-GROG can also be used to regrid undersampled non-Cartesian data, or even single points, which is not possible with convolution-based methods. Thus, SC-GROG is a promising alternative to standard gridding methods given the ubiquitous nature of mutli-channel arrays.

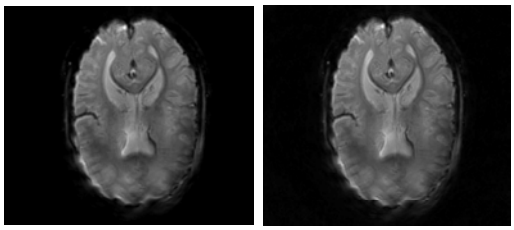

Figure 3 left: Spiral image regridded with SC-GROG.
Figure 3 right: Gold-standard convolution gridding spiral image.

References

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