Fast conjugate phase reconstruction using Taylor series approximation

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Introduction: Field inhomogeneity commonly exists in MRI, and becomes more significant at high field strength. Field inhomogeneity causes geometrical distortion and intensity variations for acquisition using Cartesian trajectories. For acquisition using non-Cartesian trajectories, such as spiral or radial trajectories, field inhomogeneity causes image blurring artifacts, which can impair the diagnostic value of the images. Conjugate phase reconstruction (1-3) and its fast approximations (4-6) are widely used methods of compensating for field inhomogeneity effects. In this abstract, we present a new fast alternative to conjugate phase reconstruction based on a Taylor series approximation of the off-resonance phase accrual term.

Theory: Ignoring T2 relaxation, conjugate phase reconstruction can be expressed as:

$$m_{CP}(\mathbf{r}) = \int_t s(t)W(t)\exp(j(2\pi\mathbf{k}(t)\cdot\mathbf{r} + \Delta\omega(\mathbf{r})t)dt \qquad [1]$$

where $s(t), \mathbf{k}(t), \Delta\omega(\mathbf{r})$, and $W(t)$ are the time signal, k-space trajectory, field inhomogeneity, and the density compensation accounting for the non-uniformity of the data sampling in k-space, respectively. Direct computation of Eq. [1] is time consuming. Fast approximations include time-segmented reconstruction (4), frequency-segmented reconstruction (5), and mutlifrequency interpolation (6). All of these methods reduce the computation cost of conjugate phase reconstruction by employing different approximations for the off-resonance phase accrual term $\exp(j\Delta\omega(\mathbf{r})t)$. A new fast alternative to the conjugate phase reconstruction can be developed by noticing that this phase term can be replaced with its Taylor expansion, which can be expressed as follows:

$$m_{CP}(\mathbf{r}) = \sum_n \frac{[j\Delta\omega(\mathbf{r})]^n}{n!} m_n(\mathbf{r}), \text{ with } m_n(\mathbf{r}) = \int_t W(t)s(t)t^n \exp[j2\pi\mathbf{k}(t)\cdot\mathbf{r}]dt, \ n = 0,1\ldots \qquad [2]$$

where $m_n(\mathbf{r})$ are referred to as base images. The base images can be reconstructed by replacing the time signal $s(t)$ with its multiplication by t^n followed by conventional gridding reconstruction. The direct calculation of the base images requires performing gridding individually for each of them, which is computationally inefficient. In our implementation, we perform gridding on the time signal $s(t)$ to acquire a Cartesian time signal, and also interpolate the time t onto a Cartesian grid using Delaunay triangulation to form a Cartesian time mask. To compute the base images, we then simply multiply the Cartesian time signal by the time mask n times and then perform a Fourier transform. Since the time mask is only a function of the k-space trajectory and sampling period, it can be computed off-line if the k-space trajectory and sampling period do not change during the scan.

The number of base images required is proportional to the off-resonance phase accrual, $\Delta\omega(\mathbf{r})t$, which can be reduced by incorporating center frequency or linear off-resonance correction (7) into the algorithm, since the bulk center/linear terms are subtracted from $\Delta\omega(\mathbf{r})$ in the phase term. Equation [2] now can be written as:

$$m_{CP}(\mathbf{r}) = \sum_n \frac{[j\Delta\tilde{\omega}(\mathbf{r})]^n}{n!} \tilde{m}_n(\mathbf{r}), \text{ with } \tilde{m}_n(\mathbf{r}) = \int_t W'(t)s'(t)t^n \exp[j2\pi\mathbf{k}'(t)\cdot\mathbf{r}]dt, \ n = 0,1\ldots \qquad [3]$$

where $\Delta\tilde{\omega}(\mathbf{r})$ is the residual off-resonance after subtracting the center/linear off-resonance frequency from $\Delta\omega(\mathbf{r})$, $s'(t)$ is the signal demodulated by the center off-resonance frequency, and $W'(t)$ and $\mathbf{k}'(t)$ are the updated density compensation function and the warped k-space trajectory, respectively, with the linear off-resonance correction incorporated.

Method and Results: We compared the accuracy of different fast conjugate phase reconstruction methods by simulation studies. The simulation was conducted at two different off-resonance frequency ranges, ±60Hz and ±100Hz. The sampling duration was 16.4ms in each case. At individual off-resonance frequencies with 2Hz increment within the off-resonance frequency range and at each sampling time point with sampling period 2 microseconds, we calculated the difference between the exact phase term $\exp(j\Delta\omega(\mathbf{r})t)$ and the approximated value from the fast conjugate phase reconstruction methods. The logarithm of the sum of the absolute value of these differences was plotted against the number of base images/segments employed in the fast conjugate phase reconstructions in Fig. 1. The simulation studies indicated that the proposed method (solid lines in Figure 1) usually requires more base images than the other fast conjugate phase reconstruction methods when the off-resonance phase accrual is relatively large. This problem is mitigated at small off-resonance phase accrual. An advantage of the proposed method indicated by the simulation is that it can ultimately achieve better approximation accuracy than all the other existing methods.

Figure 2 show an off-resonance correction example on a phantom data set acquired using spiral sequence on a 1.5 T Siemens Avanto scanner. The spiral readout was 16.4ms with 14 interleaves. A low resolution field map was acquired using two single-shot spirals to support off-resonance correction. The deviation of the off-resonance frequency from the bulk frequency is within ±30Hz and 5 base images were used accordingly. Note that the image blurring was removed after applying the proposed method. We also performed mutlifrequency interpolation (6) on the same data set. The off-resonance correction results using the two methods were not visibly different.

Discussion: The proposed method usually needs more base images than the other fast conjugate phase reconstruction methods when the off-resonance phase accrual is large. This problem is mitigated by incorporating center frequency or linear off-resonance correction to reduce the off-resonance phase accrual. The Taylor expansion can be performed around non-zero time points. Performing Taylor expansion around a non-zero time point likely can further reduce the number of base images required. T2/T2* decay was ignored in this study. The proposed method can also be applied to simultaneously compensate T2/T2* decay and off-resonance effect.

Conclusion: We proposed a new fast conjugate phase reconstruction method based on the Taylor expansion of the off-resonance phase accrual term. This algorithm achieves high approximation accuracy and can be performed rapidly with the developed computational strategies.

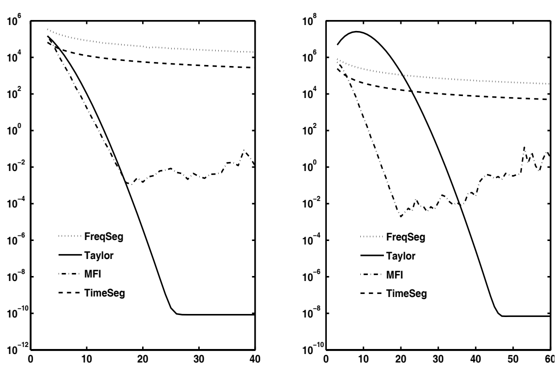

Figure 1: Semi-log plot of the approximation error (vertical axis) as a function of the number of base images/segments used (horizontal axis). The off-resonance range is ±60Hz (left) and ±100Hz (right), respectively. The sampling time duration is 16.4ms.

Reference: (1) Macovski, MRM 2, p29 (1985) (2) Norton, ITMI, MI-6, p21(1987) (3) Maeda et al, ITMI, MI-7, p26, 1985 (4) Noll et al, ITMI, 10, p629 (1991) (5) Noll PhD thesis (6) Man et al, MRM 37: 785-792 (1997) (7) Irarrazabal et al, MRM 35, 278-282 (1996)

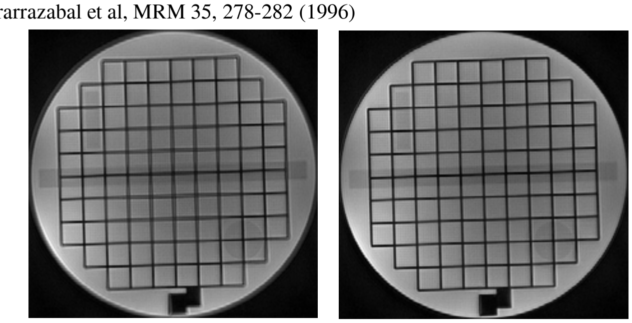

Figure 2: Left: Image acquired using spiral sequence without deblurring; Right: Image deblurred using the proposed