3D kSPA Image Reconstruction for Undersampled Arbitrary Trajectories

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INTRODUCTION: Although 3D parallel imaging is conceptually a simple extension of 2D parallel imaging, its application has been far less common than its 2D counterpart due to various technical difficulties (1). In particular, more development is required for generic algorithms that can be applied to arbitrary 3D sampling trajectories. In principle, the iterative conjugate gradient SENSE algorithm still applies in the 3D case (2). However, the computational load and memory requirement render it unpractical. Here, we present a novel 3D kSPA (k-space SPArse matrix) parallel imaging technique that works on general non-Cartesian trajectories. The kSPA algorithm computes a sparse approximate inverse of the k-space encoding matrix. Using this inverse matrix, the Fourier transform of an image can reconstructed through a matrix-vector product. This technique is demonstrated using a multi-shot 3D stack-of-spiral sequence. Excellent image quality is demonstrated with a reduction factor of 2.

METHOD: Assuming that the receiving sensitivity of the *n*-th coil has a Fourier transform of $s_n(\mathbf{k}_{\rho})$ on a Cartesian grid ($\rho = 1 \cdots N^3$ for an $N \times N \times N$ grid), the data on an arbitrary **k**-space location can be written as,

$$d_n(\mathbf{\kappa}_{\mu}) = \sum_{\rho=1}^{N^3} m(\mathbf{k}_{\rho}) \sum_{\rho=1}^{N^3} s_n(\mathbf{k}_{\rho} - \mathbf{k}_{\rho}) c(\mathbf{\kappa}_{\mu} - \mathbf{k}_{\rho}), \quad [1]$$

where $c(\mathbf{k}_{\kappa})$ is the interpolation kernel. Here, both **k** and **k** are 3D vectors. With multiple receiving coils and a number of sampling locations, Eq. [1] forms a system of linear equations that can be denoted as $\mathbf{d} = \mathbf{G} \mathbf{m}$. Here, **d** is a column vector stacked with the **k**-space data acquired by all



Fig. 1 – An illustration of 3D kSPA reconstruction for non-Cartesian trajectory. The small cube inside the stack of spiral represents the size of reconstruction kernel.

coils; **m** is also a column vector with the **k**-space value to be estimated; **G** is the coefficient matrix. The kSPA algorithm approximates the coefficient matrix with a sparse matrix and computes a sparse approximate inverse \mathbf{M}^+ such that $\mathbf{m} = M^+ \mathbf{G}^{\mathbf{H}} \mathbf{d}$.

A 3D stack-of-spiral spoiled gradient recalled echo (GRE) sequence was implemented on a 3.0T whole-body system (GE Signa, GE Healthcare, Waukesha, WI) equipped with a maximum gradient of 50mT/m and a slew rate of 150 mT/m/s. An 8-channel head coil (MRI Devices Corporation, Pewaukee, WI) was used for image acquisition. The acquisition matrix was $128 \times 128 \times 32$. The sequence parameters were: flip angle = 21° , TE = 8ms, TR = 90ms, FOV = 22cm and slice thickness = 1.0mm. For each slice encoding line (of the total 32 lines), data were acquired on a ten-interleaf spiral trajectory. A reduction factor of 2 was achieved by skipping every other interleaf.

RESULTS: Figure 2 compares the images reconstructed with kSPA (Figure 2b) with normal gridding reconstruction (Figure 2a). As expected, the gridding reconstruction results in severe aliasing artifacts due to the undersampling. Comparing to the reference images (Figure 2c), no obvious artifacts are observed in the kSPA images.

DISCUSSION: We have demonstrated that kSPA is a feasible 3D parallel imaging reconstruction method for general non-Cartesian sampling trajectories. Images reconstructed with kSPA



Fig. 2 – A representative set of images from a 128x128x32 volume reconstructed by: (a) gridding and sum-of-squares reconstruction; (b) kSPA; (c) reference images without undersampling.

from *in vivo* data acquired with a stack-of-spiral GRE sequence shows excellent quality without discernable undersampling artifact. Although examples are only shown for the spiral trajectory, this algorithm applies to arbitrary trajectories.

Such 3D parallel imaging reconstruction method not only has strong potential for standard volumetric imaging, but also has great application in dynamic imaging. 3D imaging offers higher signal-to-noise ratio (SNR). This SNR gain compensates for the SNR loss resulting from g-factor of parallel imaging. 3D acquisition, therefore offers an attractive alternative for parallel imaging. Because kSPA computes the reconstruction matrix only once, and reuses it for all images acquired in the same scan, 3D kSPA is potentially important for dynamic imaging. For example, when kSPA is combined with 3D fMRI (3), it benefits from both the SNR gain of 3D acquisition and the faster reconstruction speed of kSPA.

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REFERENCES: 1. Weiger M, et al, MAGMA, 2002;14(1);10-9. 2. Pruessmann K, et al Magn Reson Med. Magn Reson Med 2001;46(4):638-651. 3. Hu Y, Glover G. Proc 14th ISMRM, 2005.