dSENSE: direct *k*-space reconstruction for non-Cartesian parallel MR imaging

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Introduction: Recently, direct algorithms have been proposed for the reconstruction of subsampled non-Cartesian acquisitions in parallel imaging [1, 2]. These methods represent an interesting alternative to iterative algorithms [3], since they promise to be computationally more efficient whenever several images acquired with the same sensitivity encoding need to be reconstructed. We present a new, direct algorithm, which has been derived from Ref. 2. It carries out the reconstruction in *k*-space and approximates each *k*-space sample to be estimated by a linear combination of only a subset of the acquired data. The amount and the positions of the data that are included in the reconstruction of a particular *k*-space sample turn out to have a strong influence on the numerical properties and the computation time of the algorithm, as well as the achieved suppression of the subsampling artefacts. A method is discussed that ensures a good compromise between these issues.

Methods: Let $\mathbf{s}_\gamma$ be the signal in *k*-space acquired with coil element γ, (sub)sampled at arbitrary positions $\mathbf{k}_\kappa$, and m the magnetization signal in *k*-space to be reconstructed at positions $\mathbf{k}_\mu$. To avoid gridding in the final Fourier transform, the positions $\mathbf{k}_\mu$ are chosen on a regular lattice. Let W be a subset of sampled *k*-space positions located in the vicinity of $\mathbf{k}_\mu$. Denoting $\widetilde{\mathbf{C}}_{(\gamma,\kappa),\mu} = \int c_\gamma(\mathbf{r}) \exp\left(i2\pi(\mathbf{k}_\kappa - \mathbf{k}_\mu)\mathbf{r}\right) d\mathbf{r}$, the proposed dSENSE estimator reads:

$$\hat{m}(\mathbf{k}_\mu) = \widetilde{\mathbf{C}}^{\mathrm{H}}_{w,\mu}\left(\widetilde{\mathbf{C}}^{\mathrm{H}}_{w}\widetilde{\mathbf{C}}_{w} + \alpha^{-2}\Psi_w\right)^{-1}\mathbf{s}_w \; . \tag{1}$$

c_γ is the (known) sensitivity of coil element γ, $\mathbf{\Psi}$ the data noise matrix, α a regularization parameter, and the index w indicates that only positions within W are considered. The coefficients of the matrix $\widetilde{\mathbf{C}}^{\mathrm{H}}_{w}\widetilde{\mathbf{C}}_{w}$ can efficiently be computed by Fast Fourier Transforms (FFTs) and gridding, since

$$\left(\widetilde{\mathbf{C}}^{\mathrm{H}}\widetilde{\mathbf{C}}\right)_{(\gamma,\kappa),(\gamma',\kappa')} = \left(\mathrm{FT}(c_\gamma c_{\gamma'})\right)\left(\mathbf{k}_\kappa - \mathbf{k}_{\kappa'}\right) \; . \tag{2}$$

The mean square error (MSE) of the estimator is given by:

$$\mathrm{E}\left(\hat{m}(\mathbf{k}_\mu) - m(\mathbf{k}_\mu)\right)^2 = \alpha^2\left(1 - \widetilde{\mathbf{C}}^{\mathrm{H}}_{\mathrm{w},\mu}(\widetilde{\mathbf{C}}^{\mathrm{H}}_{w}\widetilde{\mathbf{C}}_{w} + \alpha^{-2}\Psi_w)^{-1}\widetilde{\mathbf{C}}_{w,\mu}\right). \tag{3}$$

For a given *k*-space sampling pattern, the MSE only depends on W. It can, therefore, be used to compare different choices of W. In practice, it has been found that, for a given cardinal number of the subset W, the condition number of the matrix $\widetilde{\mathbf{C}}^{\mathrm{H}}_{w}\widetilde{\mathbf{C}}_{w}$ and the MSE both increase in regions of high *k*-space sampling density, as in radial and spiral acquisitions in the centre of *k*-space. Based on this observation, we have designed a search algorithm that locally determines a suitable subset W with a given cardinal number. First, *k*-space is partitioned into rectangular cells of constant size, typically equal to half the spacing determined by the field of view. The Cartesian structure of this partitioning allows to rapidly find the closest cells for a given position $\mathbf{k}_\mu$, and thus the data to be included in the reconstruction. Then, to ensure computational efficiency and numerical stability in areas of dense *k*-space sampling, data samples belonging to the same cell are averaged. Eq. (2) needs to be modified accordingly, and a Taylor expansion shows that it is sufficient to evaluate the Fourier transform at a position equal to the difference between the centres of mass of the averaged data samples. The reconstruction is finally based on these averaged data, according to Eq. (1).

Radial and spiral acquisitions were simulated to assess the precision of dSENSE and to compare its results to those obtained with iterative SENSE [3]. Furthermore, dSENSE was applied to radial, real-time cardiac imaging using reduction factors from 2 to 8 and a five-element cardiac coil. The sensitivity maps were used to compute the reconstruction weights once. The frames forming the complete data set were then reconstructed simultaneously within a few seconds.

Results: Fig. 1 compares the reconstruction error of dSENSE as a function of the subset cardinal $|W|$ and of iterative SENSE for different reduction factors. A rapid convergence of the dSENSE error can be observed, indicating that subsets with small cardinal numbers are usually sufficient for the reconstruction. The remaining differences with iterative SENSE are mainly due to regularization issues and vary with the actual number of iterations used. Fig. 2 shows results from the radial cardiac imaging for a reduction factor of 8. The proposed algorithm largely suppresses the striking and approximately uniforms signal intensity over the field of view.

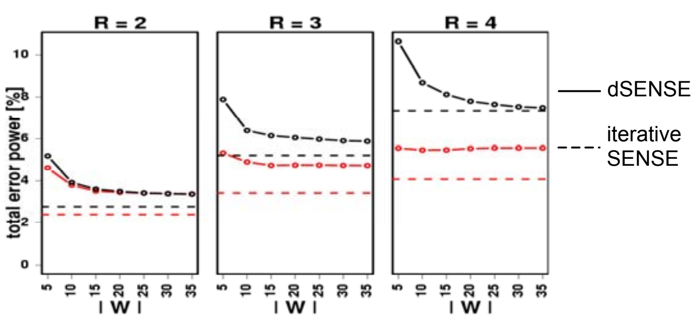

Fig.1: Reconstruction error of dSENSE and iterative SENSE for simulated radial (red) and spiral (black) acquisitions and reduction factors of 2, 3, and 4.

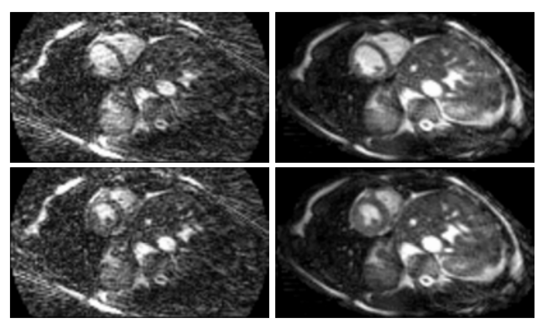

Fig. 2: Gridding reconstruction (left) and dSENSE reconstruction with $|W|$=25 (right) from an eightfold undersampled radial acquisition, for two cardiac phases.

Discussion: The proposed non-iterative reconstruction algorithm dSENSE is applicable to both Cartesian and non-Cartesian acquisitions. Its computational complexity depends only on the cardinal number of the subset used in the estimation. The mean square error turns out to be a good criterion for choosing an adequate subset. The proposed search algorithm is fast and can be applied to any *k*-space trajectory. The use of a partitioning grid enables to impose a minimal distance between averaged samples, improving the numerical stability of the reconstruction. The averaging permits to limit the size of the linear system of equations to be inverted without sacrificing SNR. With this approach, the computational complexity is reduced in comparison to other direct methods [1,2], and the quality of the reconstruction is comparable to that achieved with iterative SENSE [3].

References:
[1] Yeh et al. MRM, 53:1383-1392, 2005. [2] Sénégas et al. Proc ESMRMB, 188, 2005. [3] Pruessmann et al. MRM, 46:638-651, 2001.