Calculation of SNR degradation due to non-ideal weighting coefficients

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Introduction

k-space based parallel imaging reconstruction method like GRAPPA [1] shows very low artifact compared to image domain method. Due to the non-linear Sum-Of-Square (SOS) operation, the signal-to-noise ratio (SNR) behavior has not been analyzed in detail. In this abstract, we derive a general expression for SNR degradation due to non-ideal weighting coefficients and use this result to analyze the SNR degradation in parallel image algorithm. The SNR degradation is inversely proportional to the g-factor proposed by Pruessmann [2], but different from SENSE based algorithm, the SNR degradation in GRAPPA is image content dependent and is not only a function of coil geometry.

General SNR degradation due to non-optimum weighting coefficients

Assume we have *nCh* array coil system with noise correlation matrix given by ψ, the SNR of this array coil system with the weighting coefficients $w=(w_1, w_2, \cdots, w_{nCh})$ is given by (1), where the vector $p=(p_1, p_2, \cdots, p_{nCh})^T$ denotes the coil sensitivity. The signal and averaged noise amplitude is given by $|w \cdot p|$ and $\sqrt{w \cdot \psi \cdot w'}$ respectively. We use the Matlab notation M' for complex conjugate transpose of the matrix M.

$$SNR(w) = \frac{Signal}{Noise} = \frac{|w \cdot p|}{\sqrt{w \cdot \psi \cdot w'}} \quad (1)$$

$$SNR^{opt} = \sqrt{p' \cdot \psi^{-1} \cdot p} \quad (2)$$

As shown in [3], the maximum available SNR of (1) is reached when $w = p'\psi^{-1}$ and the corresponding maximum SNR is given by (2). The SNR degradation due to non-ideal weighting coefficient compared to the maximum available SNR can be then expressed by (3).

$$SNR^{rel}(w) = \frac{SNR(w)}{SNR^{opt}} = \frac{|w \cdot p|}{\sqrt{w \cdot \psi \cdot w'} \cdot \sqrt{p' \cdot \psi^{-1} \cdot p}} \quad (3)$$

Expression (3) can be used for any arbitrary array coil combination algorithm with the known weighting coefficients. The profile p can be based on any arbitrary profile, for instance, SOS or body coil profile. It can be proved mathematically that the reciprocal of (3) leads to the SENSE g-factor, $g = 1/SNR^{rel}(w_{SENSE})$ when SENSE weighting coefficients are used for w.

Noise amplification in GRAPPA algorithm

For GRAPPA method, it is not possible to give an expression for the weighting coefficient without image or signal content. Therefore SNR degradation can be only calculated using the equivalent weighting coefficient as presented [4]. Fig. 1 shows the phantom image acquired on the Siemens Symphony system using TSE sequence. Body coil array and spine array are placed above and below the phantoms. 189 phase encoding lines are acquired with 2x GRAPPA acceleration mode. 10 extra reference lines are acquired for coil calibration. The coil system setup is chosen so that noise amplification for GRAPPA is very strong to show the effect.

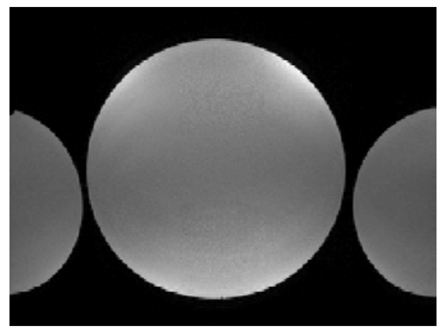

Fig. 1 Phantom images acquired with GRAPPA and body array coil

Fig.2 shows the SNR degradation due to non-optimal weighting coefficient. Compared with SENSE method, GRAPPA shows comparable SNR degradation in some region, but in some other region, the SNR degradation is higher. All the weighting coefficients are normalized to have the same reconstructed image intensity.

To understand the difference between GRAPPA and SENSE method, we compared the weighting coefficients between SENSE, GRAPPA and SOS algorithm. Fig.3 shows weighting coefficients in a small region with the strongest SENSE artifact, strong GRAPPA noise and low GRAPPA artifact. We can clearly see the SENSE coefficients are very close to SOS algorithm whereas the weighting coefficients for GRAPPA are much larger and different from SOS algorithm. The coil elements #3 and #5 with the highest weighting coefficients have very low signal amplitude, which is not optimized from the point of view of SNR. Fig. 4 shows weighting coefficient in a region where the GRAPPA SNR is almost same as SENSE. The weighting coefficient is in this case comparable.

Compared to method proposed in [5], the eq. (3) provides a more general and exact evaluation of the SNR degradation, which also avoids using time consuming statistical evaluation.

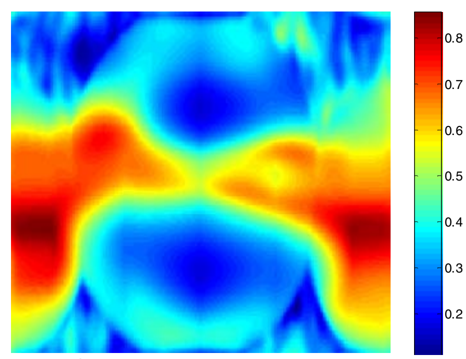

Fig. 2, SNR degradation due to parallel imaging

Discussion

Different from image domain reconstruction method where the SNR degradation is only a function of the coil geometry, the reconstruction noise of GRAPPA has a strong random character, which is also image or subject content dependent.

GRAPPA algorithm sacrifices SNR to achieve lowest artifact power and optimize SNR when artifact power is low enough.

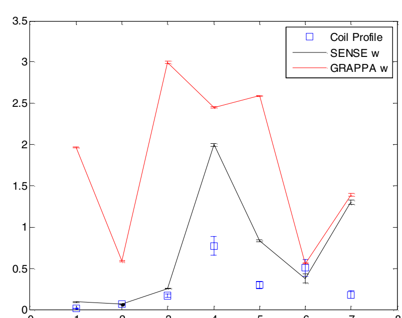

Fig. 3 High SENSE artifact, high GRAPPA noise

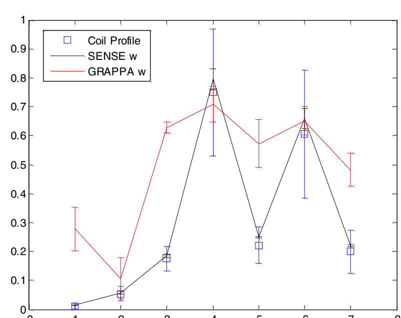

Fig. 4 Lower GRAPPA noise power

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