On Minimization of Net Lorentz Force on Gradient Coil

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Introduction The gradient coil in an MRI magnet experiences the Lorentz force resulting from the interaction of the current the coil carries and the static magnetic field generated by the magnet. The force causes the gradient coil to vibrate. The vibration generates acoustic noise and may cause image artifacts. There have been attempts to minimize the net Lorentz force on the gradient coil [1,2]. In this work, the net Lorentz force is analyzed through the theoretical treatment of a cylindrical *x* gradient coil. Some general conclusions about the minimization of the net Lorentz force on the gradient coil are presented.

Theory In what follows, we will assume the static field of the magnet is generated by the current distributed on a single cylindrical surface of radius *c*. However, the conclusions reached below are valid for the magnets whose coils are wound on multiple cylindrical surfaces. By symmetry, this static field has only the radial and axial components. In the region of $\rho < c$, they can be expressed in terms of Fourier-Bessel integrals as

$$B_\rho(\rho, z) = \frac{i\mu_0 c}{2\pi}\int_{-\infty}^{\infty} kdkJ_\phi(k)e^{ikz}I_0'(|k|\rho)K_0'(|k|c) \quad (1)$$

$$B_z(\rho, z) = -\frac{\mu_0 c}{2\pi}\int_{-\infty}^{\infty} |k|dkJ_\phi(k)e^{ikz}I_0(|k|\rho)K_0'(|k|c) \quad (2)$$

where $J_\phi(k)$ is the Fourier transform of the current density of the magnet. The current density of a cylindrical *x* gradient coil of radius *a* can be expressed in terms of its Fourier transform as

$$j_\phi^p(\phi, z) = \cos\phi \; j_\phi^p(z) = \frac{1}{\pi}\cos\phi\int_{-\infty}^{\infty} dk \; j_\phi^p(k)e^{ikz} \quad (3)$$

$$j_z^p(\phi, z) = -\frac{i}{\pi}\sin\phi\int_{-\infty}^{\infty} dk\,\frac{1}{ka}\, j_\phi^p(k)e^{ikz} \quad (4)$$

where $a < c$. The Lorentz force distributed on the surface of the gradient coil is the cross product of the gradient current density and the static field. The net force is obtained by integrating the force distribution over the surface of the gradient coil, and is only in the *x* direction, which is given by

$$F_x^p = -\mu_0 c\int_{-\infty}^{\infty} dkJ_\phi(k)\, j_\phi^p(k)|k|aI_1'(|k|a)K_0'(|k|c) \quad (5)$$

Assuming a shielding coil for the gradient coil above is located on a coaxial cylindrical surface of radius *b*, we can easily write the net force on the shielding coil as

$$F_x^s = -\mu_0 c\int_{-\infty}^{\infty} dkJ_\phi(k)\, j_\phi^s(k)|k|bI_1'(|k|b)K_0'(|k|c) \quad (6)$$

where $j_\phi^s(k)$ is the Fourier transform of the shielding current density and $a < b < c$. As a special case, if the static field is homogeneous, the net force on each of the two gradient coils is given by

$$F_x^v = \pi u B_0\int_{-\infty}^{\infty} j_\phi^v(z)dz \quad (7)$$

Here *u* and *v* are respectively *a* and *p* or *b* and *s*, and B_0 is the homogeneous static field.

Discussion Firstly, the integral in Eq. (7) is the sum of the total current over the whole range of the coil in the z direction at $\phi = 0$. It can be shown that the sum is always zero. Therefore, the gradient coil experiences no net force in a homogeneous field. Secondly, the zero net force on the shielded gradient coil set in an inhomogeneous static field can be achieved by setting $F_x^p + F_x^s = 0$. This requires

$$j_\phi^s(k) = -\frac{aI_1'(|k|a)}{bI_1'(|k|b)}\, j_\phi^p(k) \quad (8)$$

It can be seen that Eq. (8) is simply the shielding condition for the cylindrical *x* gradient coil [3]. It means that a perfectly shielded *x* gradient coil set experiences no net force. The result can be understood as follows. Since the perfectly shielded gradient coil set generates no field outside the shield, it exerts no force on the magnet. By Newton's third law of motion, the magnet should exert no force on the gradient coil set either. Therefore, in designing the shielded gradient coil set, it is not necessary to minimize the net force. It should be noted the discussion above is valid for both symmetrical and asymmetrical coils since no requirement of symmetry with respect to $z = 0$ is imposed on the gradient coil. Similar results can also be derived for the *y* and *z* gradient coils.

Conclusion It has been shown that the minimization of the net Lorentz force on a shielded gradient coil is equivalent to the optimization of the magnetic shielding. The net force on a gradient coil is smaller in a more homogeneous magnetic field.

References

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