Noise correlation matrix for multiple RF coils derived from first principles of statistical physics

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Introduction

Noise correlation has been a controversial and confusing topic in MRI literature (1,2), due to the lack of a clear definition of thermal noise and the complexity of circuitry in MR scanners. The formulation outlined in the original Nyquist paper does not provide a straightforward account for noise correlation among two circuits (3). With the widespread use of multiple rf channel technology on MR scanners, the noise correlation matrix among coil elements is becoming a very important issue for coil design (4,5), and it is desired to have a fundamental understanding of the noise correlation. Here we attempt to elucidate the noise correlation from the first principle of thermal statistical physics.

Theory

The energy stored in a system of circuits is (6), $H(q_1...q_n, i_1...i_n) = \sum_{ab}L_{ab}i_ai_b/2 + \sum_a q_a^2/2C_a$, where i_a is the current in coil a, q_a the charge, C_a the capacitance, and L_{ab} the mutual inductance between coils a and b. The thermal probability for various current values is determined by the Boltzman distribution. The noise correlation matrix is derived from the thermal average of current fluctuation (7): $\langle i_ai_b\rangle = \int i_ai_b \exp(-\sum_{cd}L_{cd}i_ci_d/2kT)di_1...di_n / \int \exp(-\sum_{cd}L_{cd}i_ci_d/2kT)di_1...di_n = kT(L^{-1})_{ab}$. Hence,

$$\Psi_{ab} \equiv \langle i_ai_b\rangle = kT(L^{-1})_{ab}. \qquad [1]$$

Eq.1 characterizes the total noise correlation. Noise correlation at a given frequency is derived from the fluctuation-dissipation theorem with $E^\omega/i\omega = f^\omega$ the "force" for current and $i\omega/Z^\omega = \alpha(\omega)$ the generalized susceptibility (7,8):

$$\Psi^\omega_{ab} \equiv \langle i^\omega_a i^\omega_b\rangle = 4kT\,\mathrm{Re}((Z^\omega)^{-1}{}_{ab}). \qquad [2]$$

Here Z^ω is the coil impedance matrix. The relation between the noise correlation over all frequencies and that at one frequency is $\Psi_{ab} = \int_0^\infty d\omega/2\pi\, \Psi^\omega_{ab}$, as derived from the Kramers-Kronig relation (7), $\alpha'(0) - \alpha'(\infty) = 2/\pi \int_0^\infty d\omega\, \alpha''(\omega)/\omega$, where α' and α'' are the real and imaginary part of α. From the definition of impedance matrix, $\alpha'(0) = 0$, $\alpha'(\infty) = -L^{-1}$, $\alpha''(\omega)/\omega = \mathrm{Re}((Z^\omega)^{-1})$. Therefore,

$$(L^{-1})_{ab} = 2/\pi \int_0^\infty d\omega\, \mathrm{Re}((Z^\omega)^{-1}{}_{ab}). \qquad [3]$$

The above theory is examplified in the case of a single coil made of R, L and C in series. $\langle (i^\omega)^2\rangle = 4kT\, R/(R^2+(\omega L-1/\omega C)^2)$, $\langle i^2\rangle = 4kT \int_0^\infty d\omega/2\pi\, R/(R^2+(\omega L-1/\omega C)^2) = kT/L$ (9).

Materials and methods

Two circular coils with 13.2 cm diameter were constructed. Experiments were performed on this coil system over a large tank of water. The spacing between the two coils was variable to allow a range of mutual inductance. The impedance matrix at various frequencies was measured using a HP network analyzer. Measurements of noise (voltage converted to current using the impedance matrix) were performed on a 1.5T GE SIGNA CV/i MR system. The RF excitation was disabled during imaging for noise collection. The mutual inductance matrix was estimated according to method by Grover (10).

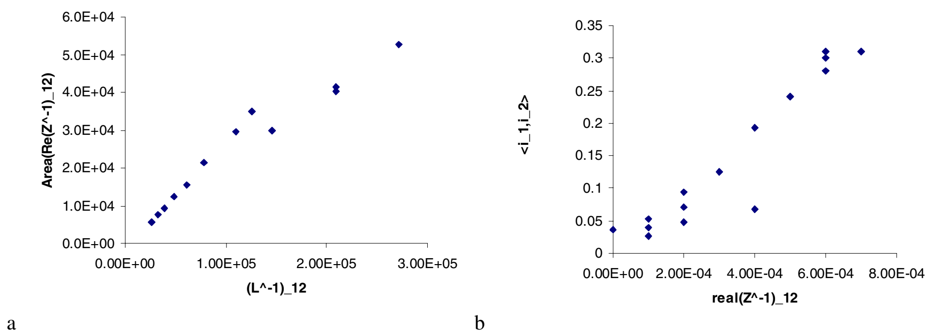

Fig. 1. a) network analyzer measured $\mathrm{Area}(\mathrm{Re}((Z^\omega)^{-1}{}_{12})) = \int_0^\infty d\omega\, \mathrm{Re}((Z^\omega)^{-1}{}_{12})$ versus inductance $(L^{-1})_{12}$, r=.95. b) MR scanner measured noise correlation versus $\mathrm{Re}((Z^\omega)^{-1}{}_{12})$, r=.94.

Results

The network analyzer measured total noise (right side of Eq.3, $\mathrm{Area}(\mathrm{Re}((Z^\omega)^{-1}{}_{12}))$)was linearly correlated to the estimated inverse inductance (left side of Eq.3, $(L^{-1})_{12}$) for the two coil system (Fig.1a), r = .95. The MR scanner measured noise matrix was linearly correlated to real part of the inverse of the coil impedance matrix (Fig.1b), r = .94.

Discussion

These preliminary experiments confirm the theoretical predictions on noise correlation (Eqs.2&3). Small errors in the experimental measurements may be attributed to weighting by the coil resonance spectrum (Q values) and effects from amplifiers.

The theory presented here clarifies confusions in MRI literature. Eq.1 indicates that the total noise correlation is zero when there is no mutual inductance, consistent with the thermodynamic argument in Ref. 1. However, the noise correlation at a given resonance frequency is determined by the impedance matrix (Eq.2), as suggested in Ref. 2. Eqs.1&2 are consistent, as indicated by the Kramers-Kronig relation Eq.3.

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