

Evaluation of Equivalent Circuit Analysis of Double-tuned RF Coils by Method of Moment

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Introduction

Double-tuned radio-frequency (RF) coils are considered essential for multinuclear magnetic resonance applications. Three-dimension numerical simulations of double-tuned coils with complicated configurations usually require sophisticated codes and very powerful computers. A simple and general equivalent circuit model is presented for analysis of both single-tuned and double-tuned RF coils. To investigate the frequency range within which the equivalent circuit analysis is effective and adequate, numerical analysis by method of moment (MOM) is employed. A double-tuned RF coil operating at ¹H/³¹P frequencies at 3 Tesla is designed and constructed. Experimental results show the agreement with theoretical analysis.

Method

Equivalent Circuit Method (ECM)

The basic principle of the ECM is to establish the equivalent lumped-circuit for an RF coil by modeling a conducting wire or strip as an inductor while ignoring all the distributed capacitances^[1]. A generalized equivalent circuit model with $N_1 \times N_2$ meshes is proposed to describe RF coils with complicated configurations in more details, even including the ending modes in the model of birdcage coils. Fig. 1 shows a part of the generalized equivalent circuit, where $L_{Xjk}^{(p,q)}$, $L_{Yjk}^{(p,q)}$ and $Z_{Xj}^{(p)}$, $Z_{Yj}^{(p)}$ are the reactive elements of the corresponding conducting paths; $I_j^{(p)}$ is the cell loop current while $I_{ring}^{(p)}$ is the ring loop current flowing in the p th path if it is equivalent to a ring. The resonant modes of the coil without any complete ring path of conduction can be easily obtained by deriving and solving the $N_1 \times N_2$ mesh equations (1), where $p=1, 2 \dots N_1$; $j=1, 2 \dots N_2$. In order to obtain all the resonant modes of the coils with ring conduction paths, one extra equation for those ring currents is put into $N_1 \times N_2$ mesh equations that is given by (2). (1) and (2) can be written in a matrix form as (3), where K, H are square matrices that depend on element parameters. Solving the eigenvalue problem derived from (3), all the resonant frequencies and the corresponding current distributions of the coil can be obtained.

$$\sum_{q=1}^{N_1} \sum_{k=1}^{N_2} i\omega [L_{Xjk}^{(p,q)} (I_k^{(q)} - I_k^{(q-1)}) + L_{Yjk}^{(p,q)} (I_k^{(q)} - I_k^{(q-1)})] + \sum_{q=1}^{N_1} \sum_{k=1}^{N_2} i\omega [L_{Xjk}^{(p,q)} (I_k^{(q)} - I_k^{(q-1)}) + L_{Yjk}^{(p,q)} (I_k^{(q)} - I_k^{(q-1)})] - Z_{Xj}^{(p)} (I_j^{(p-1)} + I_{ring}^{(p)}) + \sum_{k=1}^{N_2} i\omega (L_{Xjk}^{(p,p)} - L_{Xjk}^{(p+1,p)}) I_{ring}^{(p)} + (Z_{Xj}^{(p)} + Z_{Xj}^{(p+1)} + Z_{Yj}^{(p)} + Z_{Yj}^{(p+1)}) I_j^{(p)} - Z_{Yj+1}^{(p)} I_{j+1}^{(p)} - Z_{Yj}^{(p)} I_{j-1}^{(p)} - Z_{Xj}^{(p+1)} (I_j^{(p+1)} - I_{ring}^{(p)}) = 0 \quad (1)$$

$$\sum_{j=1}^{N_1} \sum_{k=1}^{N_2} i\omega L_{Xjk}^{(p_0,p_0)} I_{ring}^{(p_0)} + \sum_{q=1}^{N_1} \sum_{j=1}^{N_2} i\omega L_{Xjk}^{(p,q)} (I_k^{(q)} - I_k^{(q-1)}) + \sum_{j=1}^{N_2} Z_{Xj}^{(p_0)} (I_{ring}^{(p_0)} + I_j^{(p_0)} - I_j^{(p_0-1)}) = 0 \quad (2) \quad KI = \lambda HI \quad (3)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & \cdots & Z_{1,N} \\ Z_{2,1} & Z_{2,2} & \cdots & Z_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N,1} & Z_{N,2} & \cdots & Z_{N,N} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad (4) \quad I_i = \begin{bmatrix} I_1^{(i)} \\ I_2^{(i)} \\ \vdots \\ I_M^{(i)} \end{bmatrix} \quad (5) \quad V_i = \begin{bmatrix} 0 \\ \vdots \\ V_k^{(i)} \\ \vdots \\ 0 \end{bmatrix} \quad (6)$$

Method of Moment (MOM)

As a three-dimension numerical method, the MOM, taking those distributed capacitances into account, divides each wire of an RF coil along the wire's axis into a number of short segments. Each segment of the wires has its own current. The MOM equations are given by (4), where I_i is the current submatrix composed of the currents on the segments of Wire i , V_i is the exciting voltage submatrix of Wire i , and Z_{ij} is the submatrix of the mutual impedances between the segments of Wire i and those of Wire j . Assuming that Wire i is divided into M segments and excited at Segment k , I_i and V_i are then expressed as (5) and (6) respectively^[2].

Results

As an application example a four-ring birdcage coil configuration^[3] shown in Fig. 2(a) is used. The simulation programs for the coil based on ECM and MOM are developed using the MATLAB environment. According to simulation results a prototype double-tuned coil shown in Fig. 2(b) for proton (123.2MHz) and phosphor (49.6MHz) with 4 legs was constructed on a 25cm diameter Lucite cylinder using 1.3cm wide Copper tapes of 0.1mm thickness. Tuning capacitors with two different values of capacitances were used on inner and outer end rings respectively. Fig. 2(c) shows the coil's equivalent circuit.

Fig. 3 shows the dependences of the tuning capacitance value in the inner rings on the high dominant operating frequency, which are calculated from the ECM and MOM respectively. It can be seen that within the frequency range below 128MHz, the results from the ECM agree well with those of the MOM, with the largest error of 5%.

The resonant frequencies were measured by using a Hewlett Packard 4395A RF impedance analyzer. Comparison between the calculated and measured resonant frequencies of the built four-ring birdcage coil is given in Table I. The measurement results agree well with the simulations by both ECM and MOM.

Discussion and Conclusion

Evaluated by the MOM and tested by experiments on a prototype four-ring double-tuned birdcage coil, the generalized equivalent circuit model has proven precise and efficient for analysis of double-tuned RF coils within a frequency range below 128MHz. Using the generalized ECM, both single-tuned and double-tuned RF coils with various geometries, frequencies and components can be easily simulated.

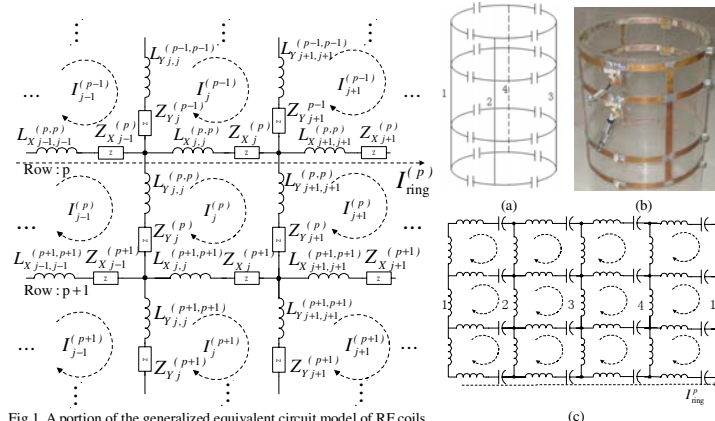


Fig.1. A portion of the generalized equivalent circuit model of RF coils

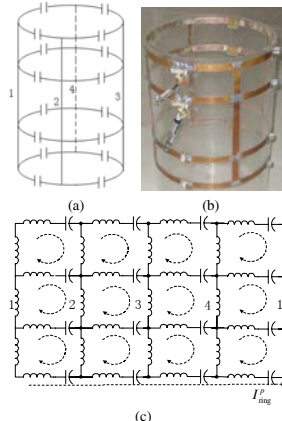


Fig.2. Coil configuration and equivalent circuit of the double-tuned four ring birdcage coil.

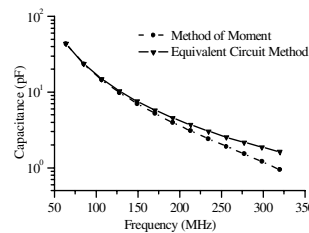


Fig.3. Tuning capacitance in the inner rings as a function of the high dominant operating frequency

Table I Resonant frequencies (MHz) of the double-tuned coil from simulations and measurements

Mode	ECM	MOM	Measurement
1 st High-order	41.95	42.20	42.03
1 st Dominant(³¹ P)	50.05	50.00	49.62
1 st End ring	73.80	73.20	70.95
2 nd End ring	77.00	76.80	74.88
2 nd High-order	101.73	99.80	97.97
2 nd Dominant	109.56	107.20	104.33
3 rd High-order	123.19	120.60	117.61
3 rd Dominant(¹ H)	130.38	127.80	123.24
3 rd End ring	131.22	128.60	123.78
4 th End ring	141.61	139.40	133.54

Reference

- [1] J. M. Boesch, et al. J. Magn. Reson., vol. 103, pp.103-114, 1994.
- [2] X. Chu, et al. Proceedings of 11th ISMRM. Toronto, 2003.
- [3] M. C. Leifer, J. Magn. Reson., vol. 124, pp. 51-60,1997.