Trajectory optimization criterion for convolution in k-t space

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<u>*Introduction*</u>: For partially acquired dynamic magnetic resonance imaging (MRI), the accuracy of the reconstruction is a function of the *k-t* space trajectory. Tsao *et al.* discussed the trajectory optimization for *k-t* SENSE [1]. *k-t* GRAPPA [2] is based on convolution in *k-t* space and may have different criteria for trajectory optimization. The intuitive basis for the optimization is that all of the data with which to construct missing data is acquired with as little redundancy as possible. In this work, the expectation of the reconstruction error is used as a criterion of trajectory optimization for reconstruction methods based on convolution in *k-t* space. *k-t* GRAPPA is applied to dynamic cardiac MRI data to confirm the criterion.

<u>*Theory*</u>: The missing *k-t* space data can be approximated by using convolution in *k-t* space. Given an acquisition trajectory, ratios between images along time direction and sensitivity maps, the true convolution matrix *M* for the location sets *p* of coil *i* at time frame *l* can be calculated by using the Eq. 1. With the reconstruction matrix, the missing data set can be calculated by using Eq 2. *k-t* GRAPPA is a special case of convolution in *k-t* space by using a truncated *M* with a small convolution kernel. Due to space limitations, the details of this theory are provided separately. Based on the theory used by Xu *et al* [3], it can be deduced that the expectation of the L_2 norm of the reconstruction error of Eq. 2 is Eq. 3, and hence expectation of the error of the reconstruction is Eq. 4. The reconstruction error expectation *E* is a function of acquisition scheme, sensitivity maps and the ratios between adjacent time frames. With a fixed system and application, the only parameter is then the acquisition scheme. If low resolution images are collected on a system for an application, then *E* can be calculated and used as a criterion to find the optimized acquisition scheme. The trajectory that generates smaller value of *E* should generate a better reconstruction. The optimized trajectory calculated on the basis of the low resolution images can then be used to acquire data for the same kind of application on the same system.

$$\overset{m(p)}{M}{}_l^i = \overset{m(p)}{C}{}_l^i \left(C_a^H \psi^{-1} C_a\right)^{-1} C_a^H \psi^{-1} \quad (1)$$

$$\overset{m(p)}{S}{}_l^i = \overset{m(p)}{M}{}_l^i \overset{a}{S} \quad (2)$$

$$E(l,i,m(p)) = tr\left(\overset{m(p)}{C}{}_l^i \left(\left(\left(\overset{a}{C}\right)^H \overset{a}{C}\right)^{-1}\right)\left(\overset{m(p)}{C}{}_l^i\right)^H\right) \quad (3)$$

$$E = \sum_{l,i,m(p)} E(l,i,m(p)) \quad (4)$$

$$RRSE = \sqrt{\sum_{domain}(recon - reference)^2 \Big/ \sum_{domain}(reference)^2} \quad (5)$$

$\overset{m(p)}{C}{}_l^i$ and *Ca* are the sub-blocks of the Fourier transform of the "scaled coil sensitivity matrix" corresponding to the missing and acquired phase encoding lines, the "scaled coil sensitivity matrix" is the pixel wise multiplication of image ratios and coil sensitivity maps; *m(p)* means the missing data at position set *p*; *l,i* are indices for time frames and channel; ψ is the noise correlation matrix among channels; *H* is the conjugate operator; *E(•)* is the expectation operator; *tr(•)* is the operator of matrix trace. It is asummed that ψ is a identity matrix in Eq. 3.

<u>*Method and Results*</u>: Axial cardiac images were collected on 1.5T GE system (GE Healthcare, Milwaukee, WI) (FOV 240 mm, matrix 192×256, TR 4530 ms, TE 1704 ms, flip angle 45°, Slice thickness 5 mm, number of averages 1) through FIESTA with a GE 4-channel cardiac coil. Breath-holds ranged from 10 - 20 seconds. There are 20 images per heartbeat. Simulated time interleaved *k-t* space data with acceleration factor 5 were used for reconstruction. Central 12 lines were used as auto-calibration signal (ACS) lines. Hence the true reduction factor was 4. *k-t* GRAPPA was used for reconstruction. Low-resolution images sequence were reconstructed using the ACS lines and some lines from adjacent time frames. The low-resolution image sequence was used to calculate the sensitivity maps and the ratios between images. The raw sensitivity maps were inpainted [4] to fill in the holes and remove noise. To reduce the amount of prescan, only the expectations (of reconstruction errors of time frame 5) were calculated by using Eq 4 with $l = 5$, which used low-resolution images of time frame 1 to 9 as prescan. Since *k-t* GRAPPA uses time-interleaved and equally spaced *k-t* space, there are 24 different trajectories. The values of *E* were calculated for all these trajectories. *k-t* GRAPPA was applied to all of these trajectories to reconstruct the images. Fig. 1 shows part of results. Root of relative square error (RRSE) is defined by Eq 5. Here the domain is the cardiac region, *i.e.* the dynamic region. It can be seen that with the increase of the value of *E*, the RRSE increased. Even though the optimization is based on reconstruction of one time frame, the optimized trajectory also generated the best results for the mean RRSE. Actually, from Figs.1, it can be seen that the optimized trajectory generates the least error for almost all time frames. The improvement can be visually seen from the difference maps (Fig. 2).

<u>*Discussion*</u>: Experimental results confirmed that the expectation of reconstruction error could be used as a criterion of trajectory optimization of convolution in *k-t* space. The optimized trajectory ([1 4 2 5 3]) for *k-t* GRAPPA is not same as the one ([1 3 5 2 4]) for *k-t* SENSE. The optimized *k-t* trajectory calculated based on one time frame can be used for other time frames. One drawback of this method is that it needs pre-scanned low-resolution images, 9 frames were used in this example. There are optimization methods independent of the application and require less computation; these methods will be discussed separately. However, application related methods would generate more accurate results since they use more information. This strategy can also be applied to GARSE [5] for trajectory optimization.

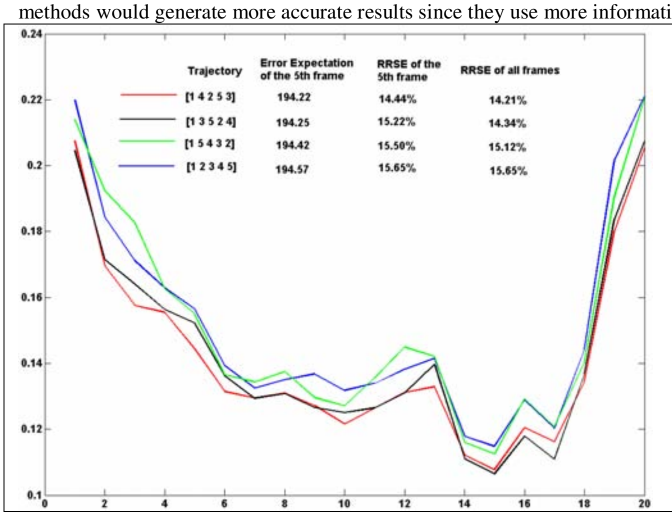

Fig. 1 The results of 4 different trajectories.

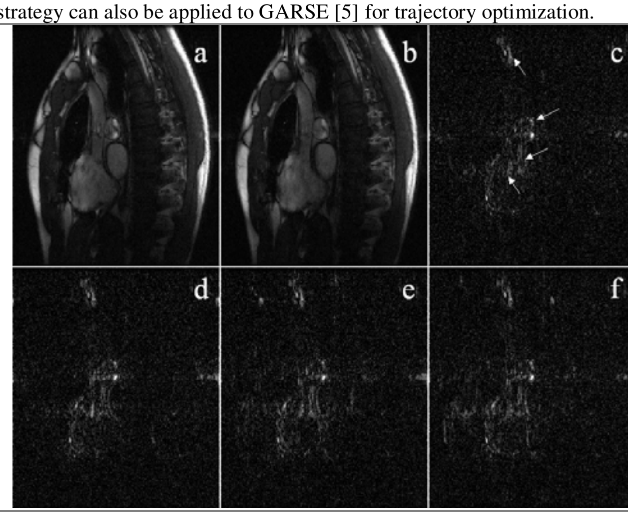

Fig. 2 The reconstructed images and the difference maps of time frame 5. a) The reference image reconstructed by using sum of squares with full *k*-space; b) The image reconstructed by *k-t* GRAPPA with trajectory [1 4 2 5 3]; The intensity scale of a) and b) is [0 5]; c) to f) are the difference maps between the reference image a) and the images reconstructed by *k-t* GRAPPA with trajectory [1 4 2 5 3], [1 3 5 2 4], [1 5 4 3 2], and [1 2 3 4 5]; The intensity scale of c) to f) is [0 1]. White arrows in c) show the locations with improved temporal resolution with the optimized trajectory.

<u>*References:*</u>

[1] Tsao J, et al. MRM 2005;53:1372 [2] Huang, F *et al.* Magn Reson Med; 54: 1172-1184,2005 [3] Xu, D *et al.* ISMRM; 2005, p2450 [4] Huang, F *et al.* Magn Reson Med; 53: 388-397,2005 [5] Kholmovski EG, *et al.* ISMRM 13, 2005, p 2257