Generalized Parameter Relations for the Shinnar-Le Roux Pulse Design Algorithm

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Introduction: The magnetization ripple amplitudes δ^e from a pulse designed by the Shinnar-Le Roux (SLR) algorithm are a non-linear function of the SLR *A* and *B* polynomial ripples δ. Pauly *et al* derived these relations for five types of pulses shown below [1]. However, these expressions do not cover the entire range of flip angles, e.g. neither the small-tip-angle or 90° relations for excitation pulses are exactly right for a flip angle of say, 70°. In deriving these relations, Pauly *et al* used a geometrical argument for excitation pulses, and a more general approach was used for other pulse types. Here, the general approach is applied to all the pulses, and gives the required general parameter relations.

Method: If the initial magnetisation is in the *z*-direction only, $M_{xy}^{+} = 2A^{*}BM_{z}^{-}$ (1), and $M_{z}^{+} = (1 - BB^{*})M_{z}^{-}$ (2), where *A* and *B* are the SLR polynomials of the rf pulse. If initial magnetisation is in the *y*-direction, after a spin-echo pulse surrounded by crusher gradients: $\left|M_{xy}^{+}\right| = \left|B^{2}M_{y}^{-}\right|$ (3). In the design process, given the desired pulse duration, slice width, and ripples in and outside the passband (δ_1 and δ_2), the optimum transition bandwidth *W* is obtained using an empirical formula (Eq. 20 in Ref. [1]). Parameters δ_1, δ_2, and *W* are then passed to the Parks-McClellan algorithm which returns an equi-ripple polynomial, which is scaled to give the *B* polynomial. This also determines the magnitude of the *A* polynomial via: $\left|A^{2}\right| + \left|B^{2}\right| = 1$ (4). Following scaling, the maximum (or minimum) and average *A* and *B* values are substituted into Eqs 1-4 and subtracted from each other to give the ripples δ^e, which can be graphed and inverted numerically to derive δ from a required δ^e.

Results: For all excitation pulses with flip angles ϕ not equal to 180°, $\left|\delta_1^e\right| = \left|\frac{2\sqrt{1-(1+\delta_1)^2\sin^2(\phi/2)}\,(1+\delta_1)\sin(\phi/2) \mp \sin\phi}{\sin\phi}\right|$ (5), and

$\left|\delta_2^e\right| = \left|\frac{2\sqrt{1-\delta_2^2\sin^2(\phi/2)}\,\delta_2\sin(\phi/2)}{\sin\phi}\right|$ (6). The sign change in (5) accounts for change from $\phi < 180°$ to $> 180°$, and expressions have been normalised to the average in-slice magnetisation. Other expressions are:

Pulse type	Parameter of interest	Pauly relations		Exact relations	
		δ_1^e	δ_2^e	δ_1^e	δ_2^e
Small-tip angles	M_{xy}	δ_1	δ_2	Eq. 5	Eq. 6
90°	M_{xy}	$2\delta_1^2$	$\sqrt{2}\delta_2$	$\sqrt{2-(1+\delta_1)^2}\,(1+\delta_1)-1$	$\sqrt{2-\delta_2^2}\,\delta_2$
Inversion	M_z	$8\delta_1$	$2\delta_2^2$	$8\delta_1/(1+\delta_1)^2$	$2\delta_2^2/(1+\delta_1)^2$
Crushed spin-echo	M_{xy}	$4\delta_1$	δ_2^2	$4\delta_1/(1+\delta_1)^2$	$\delta_2^2/(1+\delta_1)^2$
Suppression	M_z	$2\delta_1$	δ_2^2	$2\delta_1+\delta_1^2$	δ_2^2

Fig 1 shows simulated in-slice profiles from a 70° pulse designed using Eq 5 (solid), compared with those designed using Pauly's small-tip (dotted) and 90° (dashed) relations. Fig 2 shows simulated in-slice profile for 110° pulse designed with Eq 5 (solid) vs Pauly's 90° relation (dashed). Design specifications: slice width 2 kHz, ripple amplitudes = 1% for both $\delta^e_{1,2}$, and pulse duration 4 ms. Horizontal lines show the required ripple, which is achieved by the generalized parameters. The generalized parameters also gave the required ripples outside the slice (not shown).

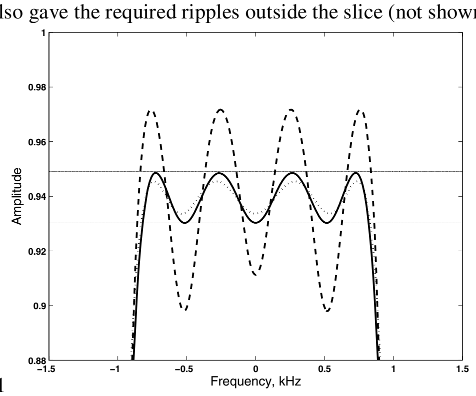

Fig 1

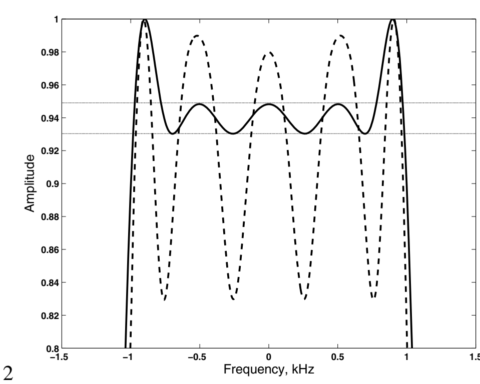

Fig 2

Discussion and Conclusion: Generalized parameter relations have been derived and show an improvement compared with the Pauly relations for flip angles not previously considered.

References: [1] Pauly J, Le Roux P, Nishimura D, Macovski A. IEEE Trans Med Imag (1991) **10**:55-65.

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