Evaluation of Conjugate-Gradient Phase Correction Algorithm under Noisy Conditions

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INTRODUCTION Self-Navigated InterLeaved Spiral (SNAILS) is a promising new diffusion-weighted imaging method which is capable of correcting motion-induced phase error by using variable-density (VD) spiral trajectories. It has been demonstrated that motion-induced phase error can be corrected by using an iterative conjugate gradient (CG) method. However, little is known about the performance of the algorithm under noisy conditions. Although the computational algorithm is similar to the original SENSE reconstruction, the phase correction problem has a unique mathematical characteristic in that each entry of the phase encoding matrix is a pure phase term. As a result, the encoding matrix is not as well conditioned and can be potentially more susceptible to physiological artifacts (noises and motions). Here we study the performance of the algorithm on simulated SNAILS data with various signal to noise ratios (SNR) and motion scales. We show that image reconstruction error is inverse proportional to SNR under typical motion levels. Additionally, low SNR may eventually cause the algorithm to diverge. However, with an empirical termination law, successful reconstruction can still be achieved for SNR as low as 5.

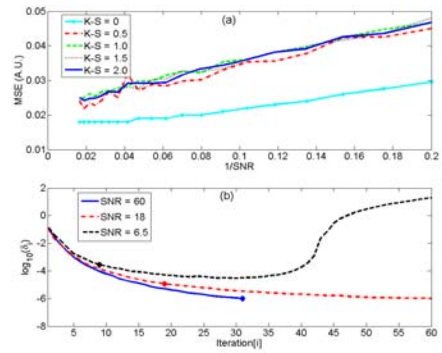

Fig 1 – Performance of SNAILS reconstruction. (a) Reconstruction errors (averaged from 50 runs) for various SNR and motion levels. (b) Log of CG criterion δ as a function of iteration count (diamonds represent the optimal termination points where the best images are obtained.)

METHODS If there is noise and motion on the subject, the samples acquired at each interleaf can be expressed as $d[k_s] = F[k_s]P_s(m+\eta_s) + n_s, n_s \sim N(0,\sigma^2)$ (1). Here, ***d*** is the vector of k-space samples and k_s denotes the non-Cartesian k-space locations at the **s**-th shot. The sampled magnetization or an $N \times N$ image is represented as a $N^2 \times 1$ vector ***m***. F and P_s represent Fourier encoding and phase encoding respectively [1]. The physiological noise on the subject is usually modeled as a Gaussian term η_i, which varies from shot to shot and is assumed to be constant within the same shot. n_s is the common thermal noise. All interleaves are concatenated and it is reformed into a positive definite (PD) problem [2], $(P^H F^H \tilde{\psi}^{-1} FP)m = P^H F^H \tilde{\psi}^{-1} d$ (2). Here $\tilde{\psi}$ is an estimated noise covariance matrix. Although the subject noise correlates at k-space samples from the same interleaf, the density weighting ***D*** used in SENSE is still a good approximation [2]. The system is thus solved in the same manner as SENSE by treating the phase encoding as sensitivity encoding.

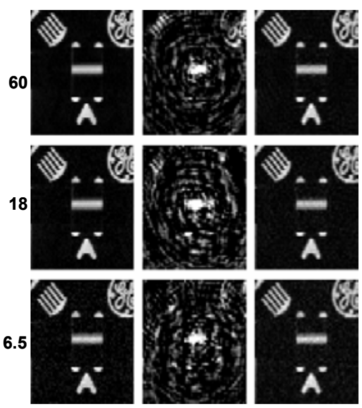

Fig 2 – Reconstruction results at different SNRs (60, 18 and 6.5). Original noise contaminated images are shown in the left the column. Images reconstructed from gridding without phase correction are shown in the middle column. And the corresponding reconstructed images by SNAILS are shown in the right.

The SNAILS algorithm is tested on 128×128 simulated images with additive Gaussian noises (only in image domain) and linear phase maps by using a 3342×8 VD-spiral trajectory. Average reconstruction errors and CG estimation errors (δ) [2] are studied at various noise and motion (k-space shift) levels.

RESULTS Fig 1a shows the mean square error (MSE) of the reconstructed image as a function of noise level (1/SNR) for each degree of motion. Fig 1b shows the evolution of CG estimation errors for typical SNR values, with diamonds marking where the best images are obtained. Reconstructed images at three SNR levels are displayed in Fig 2 for comparison.

DISCUSSION Fig 1a shows that the reconstruction error is near proportional to the noise level. Motion-induced phase degrades the quality of reconstruction but the error is not quite dependent on the degree of motion. This could be understood in a view of aliasing. Ignoring the phase encoding, the VD spiral trajectory satisfies Nyquist sampling condition in the whole k-space. However, the linear phase map on each interleaf can be considered as a k-space shift as well. The non-uniformly shifted trajectory doesn't necessarily satisfy the sampling theorem any more. Undersampling occurs only at high frequency regions for our VD spiral design, which usually doesn't distort the image quality significantly. Fig 1b shows that more iterations don't necessarily improve image quality at high noise levels and the algorithm eventually diverges when SNR is too low. However, if the algorithm is forced to terminate after only a few steps, the reconstructed image is still in reasonable quality , with a noise level comparable to that of the source image (Fig 2). This could be explained as CG searching errors. The redundant searches are mainly in noise directions when the inherent SNR is already achieved. Empirically, we terminate the CG loops when the slope of $-\log_{10}(\delta)$ drops to 1/10.

REFERENCE 1. Liu C., et al., Magn Reson Med, 52, 1388-1396, 2004; 2. Pruessmann, K.P., et al., Magn Reson Med, 46, 638-651, 2001; 3. Liu C., et al., Magn Reson Med, In Press, 2005.

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