

An Elastic Registration Algorithm based on Strain Energy Minimization and its Application to Prostate MR images

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Background

Magnetic resonance imaging using the endorectal coil (erMRI) has become the clinical standard in the diagnosis of prostate cancer as it provides excellent quality high resolution images. However the images and spectra obtained using the endorectal coil experience certain amount of displacement from their original position and the prostate is imaged in this displaced, distorted position. Such displacement and distortion during diagnosis leads to uncertainties in the localization of prostate cancer during therapeutic intervention as in the case of radiation therapy. Image registration is a necessary exercise to transform the diagnostic images to their undistorted state. Rigid body registration is inadequate since the prostate encounters non-rigid elastic deformation. Elastic registration is of particular interest in erMRI since it takes into account the physical process that prostate has experienced during the medical imaging procedure. Traditional elastic image registration schemes derive forces from image data using some similarity measure and then deform the source image into target. [1,2] Instead of computing the forces and solving the Navier-Lame equation for deformation, the proposed registration algorithm in this work models the image as a dynamic system in equilibrium and derives the deformation using the principle of strain energy minimization.

Method

Strain Energy Minimization According to the principles of dynamics, the potential energy function has a stationary value if the system is conservative and is in equilibrium. Especially, if the system is stable, then the potential energy function is minimized. [3] In prostate erMRI, the prostate is in equilibrium before and after the insertion of endorectal coil, hence the above theorem can be applied to derive the underlying deformation within the prostate.

Treating the prostate as an incompressible elastic body, the potential energy function is purely the strain energy U . It is defined as

$$U = G \iiint \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] dx dy dz + \frac{1}{2} G \iiint \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 \right] dx dy dz$$

where G is the shear modulus characterizing the prostate constitutive property, u , v , and w are displacements in x , y , and z directions, respectively. The strain energy minimization requires $\delta U = 0$. The boundary conditions of this problem are of the Dirichlet boundary form:

$$\vec{u}|_{\partial\Omega_1} = \vec{u}(u, v, w)|_{\partial\Omega_1} = 0 \text{ and } \vec{u}|_{\partial\Omega_2} = \vec{u}(u, v, w)|_{\partial\Omega_2} = (u(x, y, z), v(x, y, z), w(x, y, z))|_{\partial\Omega_2}$$

where $\partial\Omega_1$ denotes the outer boundary of the data volume and $\partial\Omega_2$ denotes the surface of the prostate.

Discretization Using the forward difference formula for first derivative of displacement, the strain energy minimization equation can be expressed in discrete form as a function of pixel positions $(X_{ijk}, Y_{ijk}, Z_{ijk})$:

$$X_{ijk} = \frac{2X_{i+1jk} + 2X_{i-1jk} + X_{ij+1k} + X_{ij-1k} + X_{ijk+1} + X_{ijk-1} + (Y_{i+1jk} - Y_{ijk}) - (Y_{i+1j-1k} - Y_{ij-1k}) + (Z_{i+1jk} - Z_{ijk}) - (Z_{i+1jk-1} - Z_{ijk-1})}{8}$$

$$Y_{ijk} = \frac{Y_{i+1jk} + Y_{i-1jk} + 2Y_{ij+1k} + 2Y_{ij-1k} + Y_{ijk+1} + Y_{ijk-1} + (Z_{ij+1k} - Z_{ijk}) - (Z_{ij+1k-1} - Z_{ijk-1}) + (X_{ij+1k} - X_{ijk}) - (X_{i-1j+1k} - X_{i-1jk})}{8}$$

$$Z_{ijk} = \frac{Z_{i+1jk} + Z_{i-1jk} + Z_{ij+1k} + Z_{ij-1k} + 2Z_{ijk+1} + 2Z_{ijk-1} + (X_{ijk+1} - X_{ijk}) - (X_{i-1jk+1} - X_{i-1jk}) + (Y_{ijk+1} - Y_{ijk}) - (Y_{ij-1k+1} - Y_{ij-1k})}{8}$$

Combining the discretized boundary conditions, the final position can be determined.

Numerical Implementation

Note, that in the above solutions, the new position $(X_{ijk}, Y_{ijk}, Z_{ijk})$ is always explicitly expressed by its closest neighbors. This particular form of solution suggests that Gauss-Seidel method can be directly used for the computation of $(X_{ijk}, Y_{ijk}, Z_{ijk})$.

Data Acquisition The prostate phantom and patient data were acquired on Philips Eclipse 1.5T system. The prostate phantom was built in-house and incorporates all the necessary elements of a prostate gland including the tissue consistency, and the biochemicals contained within a normal prostate tissue.[4] The prostate phantom also has incorporated within it seeds that serve as landmarks and are helpful in assessing the accuracy of registration. The image size of the prostate phantom was 256x256x29, and the resolution was 0.625x0.625x2.5 mm. T2-weighted images (256x256x25) of the prostate were obtained from patients with the endorectal coil in its fully inflated position and completely deflated position at a TE of 110ms and a TR of 3500ms at a voxel resolution of 0.625x0.625x3.5 mm.

Results

Figure 1 displays the representative resulting images from the registration along with the displacement vectors. The registration error was found to be 1.0 ± 0.6 pixels (or 0.6 ± 0.4 mm) for displacements ranging from 4-6mm due to the insertion of the coil. These results quantitatively demonstrated the excellent performance of the registration algorithm. Representative images from a patient's prostate are shown in figure 2. The similarity measure shows that normalized correlation coefficient improved from 0.62 ± 0.12 to 0.98 ± 0.08 by the registration.

Conclusion

In conclusion, a novel registration algorithm based on a strain energy transformation was implemented. This algorithm was tested on phantom and actual clinical data respectively. The results from phantom data showed that the accuracy of our registration is within 1 pixel. The clinical prostate data showed the feasibility of the registration algorithm to prostate imaging.

References

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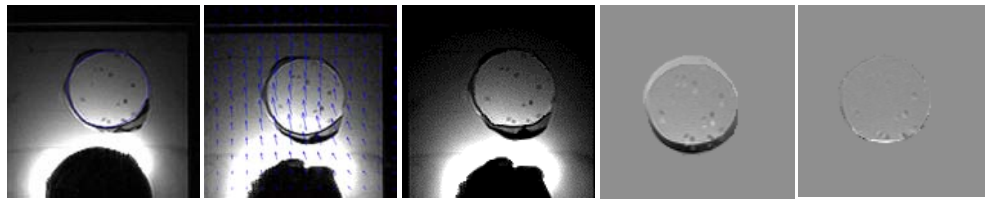


Fig. 1: Representative registration results for prostate phantom images. (a) reference image (b) source image overlapped with the displacement obtained. (c) registered image. (d) difference between the segmented reference and source. (e) difference between the segmented reference and the registered image.

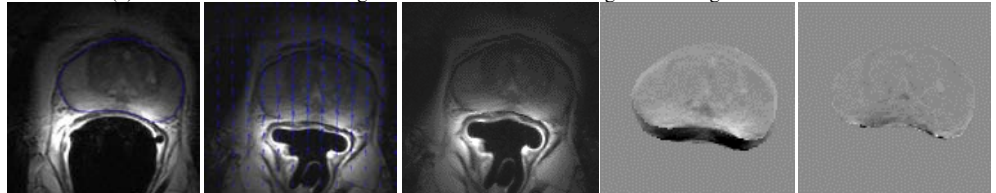


Fig. 2: Representative registration results for actual prostate images. (a) reference image (b) source image overlapped with the displacement obtained. (c) registered image. (d) difference between the segmented reference and source. (e) difference between the segmented reference and the registered image.