Autoregressive Moving Average (ARMA) Method for the Reconstruction of MR Images from Sparsely Sampled 3D k-Space

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Introduction

Magnetic resonance (MR) imaging usually reconstructs images from completely sampled but finite-sized ***k***-spaces using the inverse Fourier transform (iFT)[1]. However, iFT on finite-size ***k***-spaces introduces Gibbs ringing artifacts and degrades resolutions in the reconstructed images [2]. Moreover, real-time 3D MR imaging sometimes requires acquisition of a sparsely sampled k-space dataset [3]. For example, in continuously moving-table contrast-enhanced MR angiography sparse sampling is a necessity to ensure tracking of the contrast bolus. With sparse sampling, only a portion of the phase-encoded plane is sampled (with acquisition typically of the central region favored over the periphery). Autoregressive moving-average (ARMA) methods have been shown to extrapolate data beyond finite-sized ***k***-space datasets [4], and may be of use when reconstructing sparsely sampled 3D k-space data. Our hypothesis is that ARMA methods have the ability to recover missing data and thus generate images with improved resolution and reduced artifact, compared to zero-filling algorithms (ZF; *c.f.*, Ref [5]).

Theory

The ARMA method is a parametric approach that can be used to model (characterize) a finite-length data sequence. In signal processing terms, the measured ***k***-space data are regarded as a subset of the transient response of an infinite impulse response (IIR) filter excited by excitation pulses. Data points, S_n, can be modeled and expressed as a linear combination of their previous values, S_{n-m} ($1 \le m \le p$), and the excitation, e_{n-h} ($1 \le h \le q$), *i.e.*, $S_n = -\sum a_m S_{n-m} + \sum b_h e_{n-h}$, where p and q are the autoregressive and moving-average filter orders, respectively. Of the algorithms used to successfully implement the ARMA method, the Transient Error Reconstruction Approach (TERA) is of particular interest [4]. We have extended the regular TERA algorithm (rTERA) into a new method where the image phase is used as a constraint, *i.e.*, constrained-phase TERA algorithm (CP-TERA). However, because it is not possible to obtain the true image phase $\Phi_0(r)$, image phase from the central zone of ***k***-space $\Phi_c(r)$ could be used as an estimate of the true phase: $\Phi_0(r)$ contains mostly low spatial frequency components and thus $\Phi_c(r)$ can be considered as a good representation of $\Phi_0(r)$. After parameters a_m, b_h and e are determined from the measured MR data and the constraints, missing data would be recovered and the image function can be calculated explicitly.

Method

Raw data from a quality-control phantom were acquired on a clinical 3 T scanner (Signa; General Electric Healthcare, Waukesha, WI). Hybrid ***k***-space data (x-k_y-k_z with $N_x = 256$, $N_y = 256$ and $N_z = 64$) were produced by taking the iFT of each readout (*i.e.*, in the x-direction) immediately after acquisition and then placing them into the appropriate location in the hybrid space. A series of simulated sparsely sampled ***k***-spaces were generated (MATLAB, version 6.5.0, R13; Mathworks, Natick, MA) with constant central zone ratio α (defined as $\alpha = N_{central} / N_o$, where $N_{central}$ and N_o are the number of pixels in the central zone and in the full phase-encoded plane, respectively) and varying the sparsely sampled density β (defined as $\beta = N_{acq}/(N_o - N_{central})$, where N_{acq} is the number of points in peripheral region). Images were reconstructed from the fully sampled hybrid ***k***-space (*i.e.*, the true image I_0) and from simulated sparsely sampled ***k***-spaces using (a) ZF (I_{ZF}), (b) rTERA (I_{rTERA}) and (c) CP-TERA ($I_{CP\text{-}TERA}$). The ZF method replaces the missing ***k***-space data with zeros. The quality of the resulting images was assessed by visual inspection and quantified by calculation of performance error (PE) defined as PE = $\sqrt{\sum(\zeta_i - o_i)^2} \Big/ \sqrt{\sum o_i^2}$ where ζ_i and o_i denote pixels from I_{TERA} (or I_{ZF}) and I_0, respectively. The PE summation was performed over all pixels in the image (global performance error, GPE) and over all 3 × 3 kernels in a small high-frequency region (local performance error, LPE).

Results

Both TERA approaches produced images that were qualitatively and quantitatively superior to those obtained by ZF. CP-TERA was better able to recover the missing ***k***-space data points than rTERA (Figs 1d vs 1c). Hence, CP-TERA (Fig 1h) showed better image reconstruction performance than both ZF (Fig 1f) and rTERA (Fig 1g). Global and local PE (GPE, LPE) for CP-TERA images (Fig 2) were smaller than those when using ZF and rTERA, indicating superior image reconstruction performance with sparsely sampled ***k***-space by our proposed algorithm.

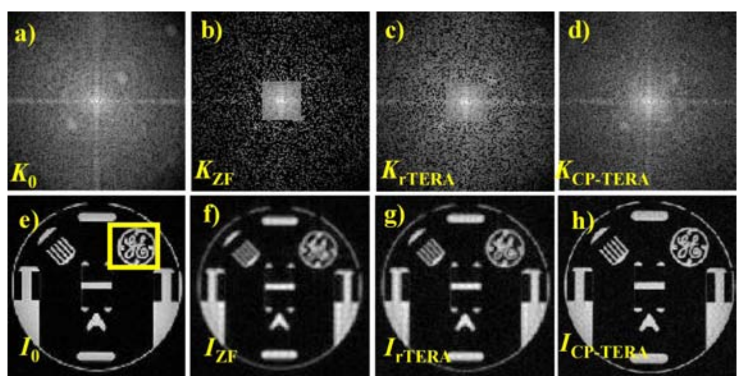

Fig 1: 2D representation of 3D ***k****-space datasets and the corresponding reconstructed images. (a) Completely sampled* ***k****-space and (e) the reference image; (b) Sparsely sampled* ***k****-space with α = 6.25% and β = 30, and (f) ZF image, (c) partially recovered* ***k****-space and (g) the corresponding image by rTERA and (d) partially recovered* ***k****-space and (h) the corresponding image by CP-TERA. Yellow outlined region in (e) used for local performance error calculation.*

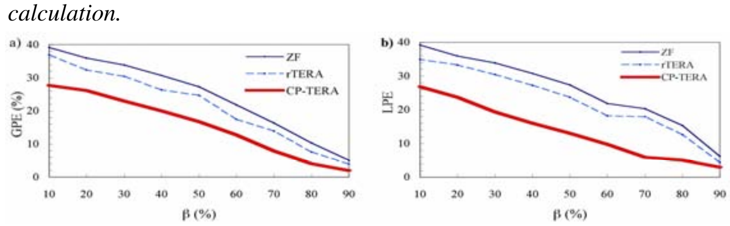

Fig 2: Global (a) and local (b) performance error (PE) versus β (using α = 6.25%) for ZF, rTERA and CP-TERA images. Local PE is defined over the yellow region shown in Fig 1e in order to highlight ability to reproduce high-frequency structures.

Discussion

Compared with the more simple and commonly used ZF reconstruction and with the rTERA, CP-TERA is a better technique to reconstruct images from sparsely sampled ***k***-space, as it results in good image quality and reduced global and local PE. This is an interesting finding demonstrating that TERA successfully interpolates the missing data. Optimization of CP-TERA to achieve computational efficiency is our next step.

References [1] Kumar, *et al. JMRI* 1975; **18**: 69-83. [2] Wood, *et al. MRM* 1985; **2**: 517-526. [3] Sabati, *et al. PMB* 2003; **48**: 2739-2752. [4] Smith, *et al. MRM* 1986; **4**: 257-261. [5] Bernstein, *et al. JMRI* 2001; **14**: 270-280.