FE-MRI: Simulation of MRI using arbitrary finite elements

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Introduction: Simulations of MRI typically rely on analytic representations of geometric primitives or voxel-based descriptions of objects derived from previously acquired high-resolution images (e.g. [1]). An alternative was proposed by [2], in which a complex object is decomposed into contiguous linear tetrahedral finite elements. This allows the MR signal equation to be integrated analytically over each element, and the resulting *k*-space contributions from all elements are used to reconstruct the simulated MR image. However, because this analytic approach is inherently limited to linear tetrahedra, many elements are still required to accurately describe realistic objects having curved or irregular boundaries. Thus we propose here the use of numerical rather than analytic integration of the signal equation, which permits a more efficient description of complex objects via meshes of higher order finite elements.

Methods: By way of demonstration, we focus here on the case of a uniform cylinder of unit diameter and height. Per Figure 1, the cylinder was discretized into *N* discrete prismatic sectors, each comprised of three linear or quadratic tetrahedra. A 2D acquisition of unit slice thickness, 64x64 acquisition matrix and in-plane resolution of approximately 20 pixels across the diameter was simulated. Numerical integration of the signal equation over all points in k-space was carried out for the quadratic mesh using tensor product 1D gauss quadrature rules within the open-source *libMesh* finite element library [3]. Numerical integration error was minimized by using a conservative polynomial integration order of 32, which served to highlight errors due to geometric discretization alone. Simulations for linear meshes were carried out via analytic integration of the signal equation within the open source *GINAC* symbolic math library [4]. Errors in the resulting simulated images were quantified as the maximum error in image intensity, *E*, relative to truth, namely the image derived from a highly-refined cylinder mesh (*N*=4096).

Results: Images generated for N=8 using linear and quadratic elements are shown in Figure 1 (bottom) to illustrate artifacts, both at the boundary and within the cylinder, due to object discretization. Figure 2 demonstrates the superior accuracy of quadratic vs. linear meshes. For example, accuracy to only two decimal places (i.e. E~0.005) is achieved with 21 quadratic elements (N=7) vs. 192 (N=64) linear elements. Even after calibrating for the fact that, for a given N, quadratic meshes require roughly three times more storage compared to linear meshes, the former remain a more efficient means of describing the object. Preliminary results also show that CPU performance is similar or better for equivalent levels of accuracy, despite the fact that numerical integration is slowed by unavoidable housekeeping routines within the *libMesh* library.

Conclusions: These results suggest that the use of superlinear elements and numerical integration can provide a significant computational gain for MRI simulation compared to analytic integration over linear tetrahedra. Although shown here for quadratic tetrahedra, the approach is trivial to extend to prismatic, hexahedral or other finite element shapes of quadratic or higher order, potentially providing further storage and speed gains. Additional gains in CPU performance are expected through the use of routines independent of the *libMesh* library, and also by varying numerical integration order with spatial frequency (i.e., k-space location). Ultimately, our goal is efficient simulation of cardiovascular MRI using compact computational phantoms having realistic structure and function.

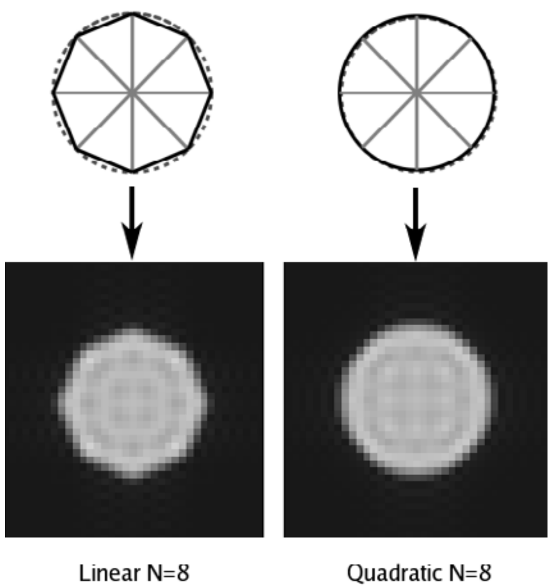

Figure 1: (Top) N=8 cylinder discretization using linear and quadratic elements. (Bottom) Resulting simulated MR images.

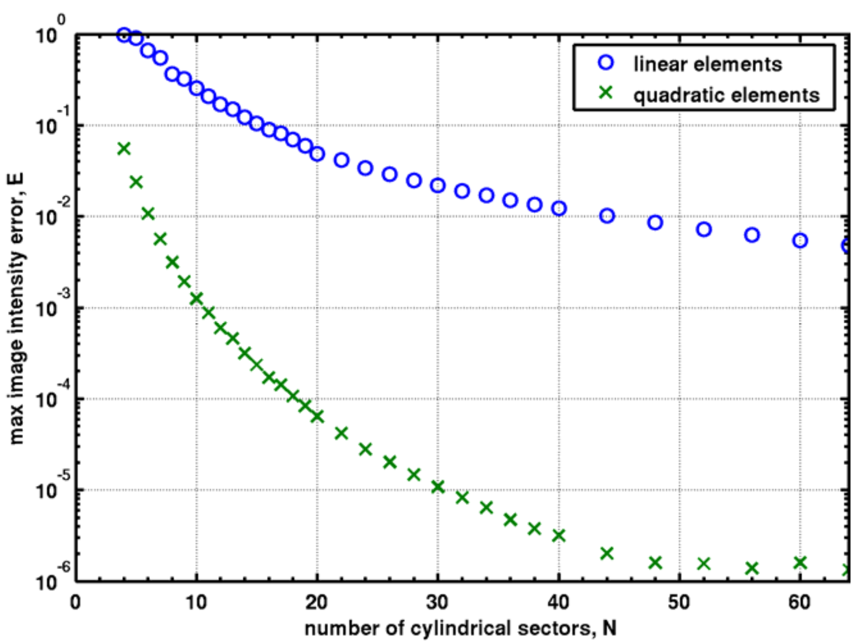

Figure 2: Effect of discretization on the simulated images.

References

[1] Benoit-Cattin H et al., The SIMRI project: a versatile and interactive MRI simulator. *JMR* 2005; 173:97-115

[2] Truscott KJ and Buonocore MH, Simulation of tagged MR images with linear tetrahedral solid elements. *JMRI* 2001; 14(3):336-40.

[3] http://libmesh.sourceforge.net/

[4] http://www.ginac.de