Improved Conjugate-Gradient Phase Correction Algorithm for Multi-Shot DWI

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INTRODUCTION Self-Navigated InterLeaved Spiral (SNAILS) image reconstruction algorithm is adapted from SENSE using a conjugate gradient (CG) method, by treating the phase correction map of each interleaf as the coil sensitivity. However, the system is not as well conditioned with each entry of the phase encoding matrix as a pure phase term. The computational errors from gridding could slow the convergence of CG and degrades the quality of reconstruction. Here, we present an improved CG algorithm (ICG) which converges much faster and improves reconstruction by introducing an additional phase correction stage. We evaluate the improved algorithm at various degrees of motion and a wide range of signal to noise ratios (SNR). Through simulation, we demonstrate that the improvement in image quality is nearly independent of SNR, while the improvement in the converging speed is more significant at higher SNR.

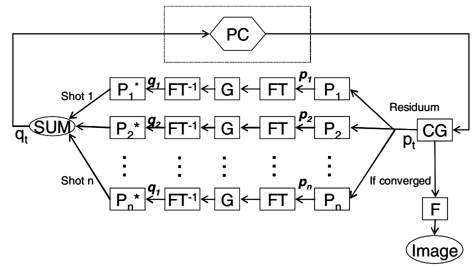

Fig 1 – Flow chart of SNAILS reconstruction. An additional phase correction stage **PC** (real part extraction) is introduced to improve the algorithm.

METHODS To use a CG solver for phase correction, the system is assumed to be symmetric positive definite (PD). The iterative phase correction process is the same as Pruessmann's algorithm [2] if the motion-induced phase error is treated as pseudo-sensitivity. The flow chart is shown in Figure 1. A pair of conjugate gridding transforms is first performed in each iteration step [1], $q = (P^H F^H F P) p$ (1)

Here, F is the Fourier encoding matrix and P is the phase encoding matrix. The Fourier transform is efficiently computed by using gridding and FFT for each interleaf, which usually produces computational errors, especially when the k-space trajectory undergoes undersampling. Then the distortion on q can be expressed as $q = (P_e P^H F^H F P) p$ (2).

The PD condition of the system is spoiled, causing a deviation in the next search. This may not be a serious problem for traditional SENSE reconstruction with near-orthogonal sensitivity encoding. However, SNAILS reconstruction is very sensitive to this error, especially to the phase error, since every entry of the phase encoding matrix is also a pure phase term. The convergence of the algorithm is thus slowed and the stability could be ruined at large noise levels.

It is difficult to compensate this phase error directly. In SNAILS reconstruction, only a real image is usually expected since the inherent image phase is also estimated into the phase encoding matrix. We can thus avoid the problem of phase dispersion and improve the convergence by forcing the CG search only in real domain. We present a very simple approach here, in which the initial guess image is forced to be real and only the real part of q is extracted at the end of each iteration (indicated by PC in Figure 1).

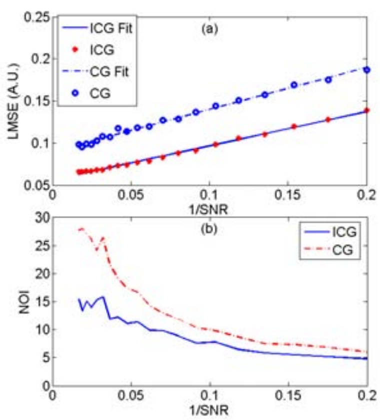

Fig 2 – Comparison of ICG and the original CG algorithm. (a) Mean square error of the reconstructed image as a function of 1/SNR. (b) Number of iterations needed at different noise levels. (More iterations are needed for high SNR to trade higher accuracy. NOI is smaller for low SNR since the residual noise error can't be further reduced after a few effective steps.)

The improved CG algorithm was tested on both 128x128 and 256x256 simulated data by using variable-density-spiral trajectory. The performance was also compared with that of the original algorithm (Fig 2). We limit the maximum number of CG iterations to be 60 and the CG stopping error as 10^{-7}. Both reconstruction algorithms are run on 100 simulated images at each noise level with various degrees of motion-induced phase. The approximate image with the least mean square error (LMSE) is selected as final reconstruction. Number of iterations (NOI) needed to obtain this image is recorded to measure the converging rate.

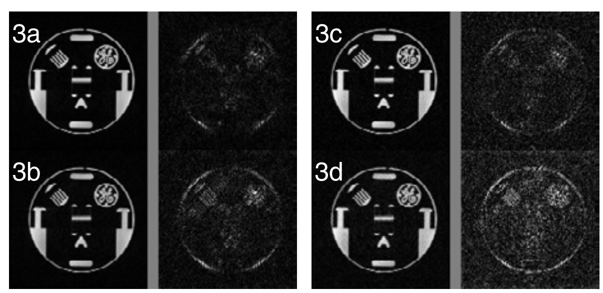

Fig 3 – Images reconstructed from ICG and CG.. The exaggerated error image is also displayed for each case. 3a) and 3b) are reconstructed by ICG and CG respectively at SNR=50; 3c) and 3d) are reconstructed at SNR=8.

RESULTS Figure 2 shows the comparison of their performance. Average LMSE and NOI are plotted as a function of noise level (1/SNR). ICG converges much faster (Fig 2b) and produces smaller reconstruction errors (Fig 2a). The LMSE is nearly linear with respect to the image noise level and the improvement is almost independent of it. The NOI is usually reduced by 20% to 50% with the PC stage applied. Figure 3 compares the reconstructed images by both algorithms. The quality of reconstruction is always improved by using ICG.

DISCUSSION We have shown that our new PC stage significantly improves the original SNAILS CG algorithm both in speed and in quality. The improvements in converging rate can be explained in a view of system scale. To do a SENSE reconstruction of an $N \times N$ image, a $(2N^2) \times (2N^2)$ real system needs to be solved if the image vector is decomposed into real and imaginary parts. By restricting real updates in CG, the system size is equivalently reduced to $N^2 \times N^2$. We also tested the algorithm by reconstructing an image from samples on a conventional spiral trajectory with known motion-induced phase maps. The original CG algorithm could not eliminate the aliasing on the final image due to severe undersampling of each interleaf at the center of k-space. Whereas ICG still does a good reconstruction.

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