# Noise Characteristics in POCS (Projection Onto Convex Sets)-Reconstructed MR Images

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**Introduction:** Undersampling of the **k**-space data has been widely used as an effective approach for rapid 2D and 3D MR imaging. Typically, the central zone of **k**-space is fully sampled but the peripheral zone is partially sampled. The partial Fourier method, [1,2] for example, reduces the amount of acquired data by asymmetrically truncating the peripheral portions of **k**-space in either the phase [1] or the frequency [2] encoding directions, or both. Projection-onto-convex-sets (POCS), [3-6] techniques have been successful in reconstructing such undersampled **k**-space data. In evaluating the noise properties of the POCS-reconstructed images and comparing them to those obtained in other reconstruction techniques (via SNR and CNR calculations), care is warranted because the iterative non-linear POCS reconstruction method can modify the underlying noise properties of the image. This preliminary study investigates and characterizes the noise in the POCS reconstruction approach.

**Materials and Methods:** Three fully sampled 3D **k**-space data sets were acquired on a clinical 3.0T MR scanner (Signa; General Electric Healthcare, Waukesha, WI) using a QA phantom and the legs and head of a volunteer. A gradient-recalled echo sequence (TR/TE/flip = 8.1 ms/ 3.2 ms/ 60°; 2-mm slice thickness; 20-cm FOV with 256 × 256 × 64 acquisition matrix) was used to image the phantom. A balanced steady-state free precession sequence (TR/TE/flip = 8.1 ms/ 4.05 ms/ 45°; 2-mm slice thickness; 20-cm FOV with 256× 256× 48 acquisition) was used to scan the legs. A time-of-flight angiography sequence (TR/TE/flip = 24 ms/ 3.2 ms/ 45°; 2-mm slice thickness, 20-cm FOV with 256 × 256 × 64 acquisition) was used to scan the head. This variety of pulse sequences was used to test the sequence dependency of the results. A commercial program (MATLAB, v6.5.0, R13; Mathworks, Natick, MA) was used to emulate undersampling and to implement the POCS reconstruction algorithm. From each fully sampled **k**-space data, we only retained a fraction of the total acquired data ( $0 \le C + P \le 1$), where $C$ (ranged from 1.25% to 6.25%) of the data retained in the fully sampled central ($k_y$, $k_z$)-phase-encoding **k**-space zone and $P$ (ranged from 10% to 85%) of the data retained in the peripheral zone. The remainder fraction, $1 - C - P$, of the **k**-space data was set to zero. POCS and zero-filling (ZF) algorithms were performed to reconstruct 3D MR images from these undersampled **k**-space data. [6] The magnitude of 10,000 randomly selected image data points from air (*i.e.*, a noise-only region) in each 3D image were statistically analyzed. Probability plots with Anderson-Darling (AD) goodness-of-fit statistic and associated $p$-values at a 95% confidence interval (CI) were employed to examine if the noise signal in the images was Rayleigh-distributed (*i.e.*, resulted from normally-distributed complex signal with zero mean); and if not Rayleigh-distributed, then the suitability of other distributions were evaluated. A commercial statistical program (Minitab, R14; Minitab Inc., State College, PA) was used for statistical analysis and for parameter estimation of the noise distribution model in POCS and ZF images. Ten widely used random distributions were assessed, including: 2- & 3-parameter gamma, Rayleigh, 2- & 3-parameter Weibull, 2- & 3-parameter lognormal, logistic, 2- & 3-parameter loglogistic, exponential, and normal distributions.

**Results:** The Figure shows probability plots of three distribution models for two ($C$, $P$)-values of POCS-reconstructed images of the legs (first panel). The lognormal plot matches the image noise distribution very well in the case of highly undersampled **k**-spaces. The AD statistic and associated $p$-values were 4.7 and > 0.05 (*i.e.*, not significantly different), respectively. For this particular dataset, the location and scale parameters of lognormal distribution were estimated to be 7.7 and 0.88, respectively. Similar plots and AD values (1.2 – 16.0) were obtained from the phantom and the head data sets. Other random distribution models showed higher deviations in their probability plots from the noise data with higher AD values and lower $p$-values (see second row in Figure for normal distribution). No statistically significant difference in modeling the noise was seen between the use of different pulse sequences and/or object type. For the ratio of acquired **k**-space data less than ~65 %, the lognormal distribution remained valid. For acquired data ratios of more than 80 %, the noise in the POCS image could be represented with a Rayleigh distribution with high confidence (> 95%). ZF-reconstructed images possessed Rayleigh distribution for noise for all ($C$,$P$)-values (not shown).

**Discussion:** In this work we demonstrated that the noise in POCS-reconstructed images can no longer be normal owing to the nonlinear processing of the image reconstruction algorithm, as opposed to linear ZF reconstruction process. Therefore, one should be careful in calculating and/or reporting noise-related parameters of such images. For POCS images from sparsely sampled **k**-space data, the lognormal distribution was shown to be a good model. As the ratio of the missing **k**-space data decreases, the noise distribution in POCS images tends to be Rayleigh-distributed. This is because most of the noise power is governed by the predominant Gaussian-distributed noise from the object. The statistical method presented here can also be used in assessing other non-linear reconstruction algorithms.

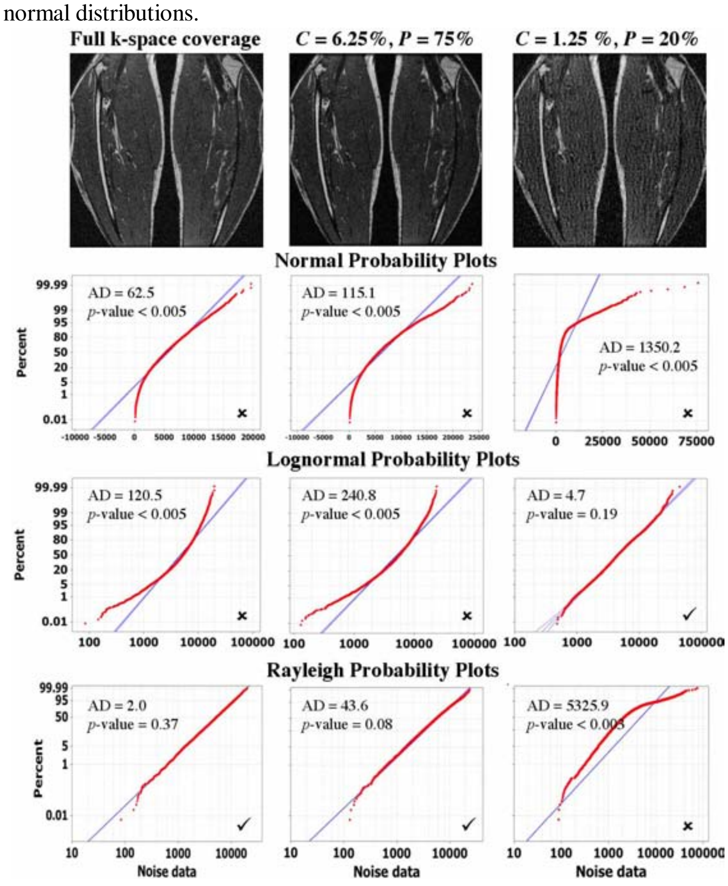


*Figure: Modeling noise in POCS-reconstructed images. Panel 1 illustrates middle slice POCS images with different k-space coverage (100%, 81.25%, and 21.25%). Panels 2-4 show three goodness-of-fit plots and parameters with a 95 % CI. The Rayleigh distribution fails to model the noise (✗) in the case of highly undersampled k-spaces. Lognormal distribution fit the noise properties well (✓) when the k-space coverage less than ~ 65%.*

**References:**

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