Colored Noise and Effective Resolution: Data considerations for non-uniform k-space sampling reconstructions

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Introduction: Non-Cartesian k-space acquisitions offer a number of advantages over Cartesian acquisitions. Often, these trajectories are designed to repeatedly sample the center of k-space. Some acquisitions, such as PROPELLER [1] or variable-density spirals [2], greatly oversample a region around the center of k-space, and thus have a great variation in the sampling density from the center of k-space to the periphery. However, oversampling near the origin of k-space increases the SNR near the center of k-space. Therefore, noise in the reconstructed image is not white, but is greater in the higher spatial frequencies. The result is colored noise in the images: loss of SNR near structures and changes in contrast. The effective resolution of an image is thus compromised. Although the noise performance of the gridding algorithm has been studied [4], the subject of noise coloration from certain trajectories seems untreated in the MR literature.

Materials/Methods: Assuming the noise in a set of acquired k-space samples is uncorrelated between samples and is zero-mean Gaussian distributed, one may define the variance of the noise in any sampled datapoint as σ^2_{est}. This value may be estimated by, for example, computing the variance of sample points k_s at the edges of k-space, where noise dominates the signal. The noise variance σ^2_g at a Cartesian-gridded datapoint may not be identical to that of the sampled data, as gridded datapoints, k_g, contain contributions from a variable number of weighted sample points, k_s. From the gridding operation [3], $k_g(x,y) = \frac{1}{D(x,y)} \sum_{x',y'\in N} k_s(x',y') \cdot c(x-x', y-y')$, where $D(x,y) = \sum_{x',y'\in N} c(x-x', y-y')$ is the local sampling density, N is the gridding window neighborhood, and c is the evaluated convolution kernel function. The variance of the gridded sample point $k_g(x,y)$, $\sigma^2_g(x,y)$ is then defined by:

$$\sigma_g^2(x,y) = \frac{1}{D^2(x,y)} \sum_{x',y'\in N} \sigma^2_{est}(x',y') \cdot c^2(x-x', y-y') = \frac{\sigma^2_{est}}{D^2(x,y)} \sum_{x',y'\in N} c^2(x-x', y-$$

In order to correct the noise coloring, noise with variance $\sigma^2_{corr}(x,y)$ must be added to any gridded points with smaller variances:

$$\sigma^2_{corr}(x,y) = \sigma^2_{est} - \sigma^2_g(x,y) = \sigma^2_{est}\left[1 - \frac{\sum_{x',y'\in N} c^2(x-x', y-y')}{\left(\sum_{x',y'\in N} c(x-x', y-y')\right)^2}\right]$$

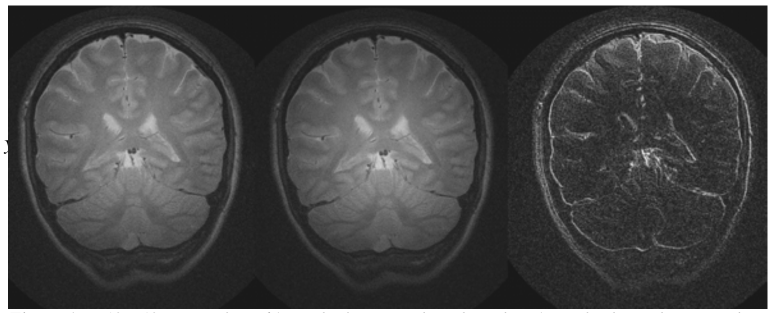

Figure 1 – 512x512 images from 64 interleaf VD spirals with pitch = 4. (Left) shows the original regridded image,(Center) shows the regridded image with the noise in each regridded point equalized across all spatial frequencies. (Right) shows the absolute difference image, which is large in areas of high frequency. SNR of the left and center images are 20.55 and 20.46, respectively.

This final bracketed term is always less than 1. The term weights the added noise's variance such that larger variate noise is added to gridded values which have more sampled points contributing to their values. Gridded data points which only have a single sample point contributing will have therefore zero additional noise added.

A 512x512 dataset was inverse gridded to a 64-interleaf VD spiral trajectory with pitch factor of 4, with 4972 points sampled in each spiral arm. Regridding was then performed back to a Cartesian k-space, both with and without noise equalization as described above, before final inverse Fourier transformation.

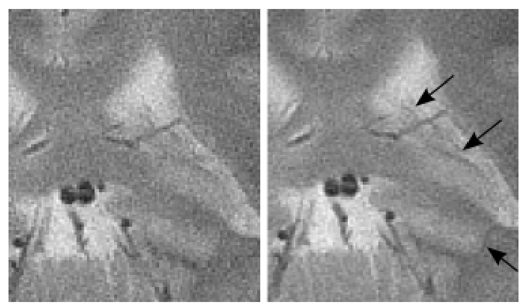

***Figure 2** – Native resolution of images in Fig. 1. (Left) shows the colored noise regrid, and (Right) shows the image after re-colored noise regridding. Increased perspicuity in fine structures is noted, as well as a less clumped noise appearance in smooth areas.*

Results: The trajectory chosen samples the center of k-space approximately 400 times more often than the least dense locations in the periphery. Fig. 1 shows the resulting images from regridding with and without noise equalization, and an absolute difference image. At this resolution, images appear quite similar, until the difference is computed. The images are most dissimilar in areas of contrast changes, as expected from the increased noise at discontinuities. Overall SNR in each image, computed as the mean in the signal area over the standard deviation in the air space, is approximately equivalent in each. Displaying a portion of each image in its native resolution in Fig. 2 allows the differences to be appreciated. The un-colored noise image on the right has better fine structure detail, and the noise in areas of even intensity is less conspicuous.

Discussion: Although noise remains zero mean Gaussian through the regridding process, the variance of this noise is not necessarily equivalent at all spatial frequencies. The coloring of noise in a reconstructed image affects the perceived resolution of the image. In order to correct this coloring, the SNR must be equalized for all spatial frequencies. In this work, a formula was derived to re-noise gridded datapoints with greater SNR than the sampled datapoints. While an additive noise field weighted by the density of samples could be used to de-color the noise, this would reduce the overall image's SNR. This method only recolors the noise, equalizing the SNR across spatial frequencies. No noise is added to gridded samples with the original sample points' SNR. While this does remove one perceived advantage of these acquisitions, higher energy per sampling time, it does not remove other advantages, such as the ability to perform navigation and motion correction, or flow and motion insensitivity through sample averaging. Using a reconstruction method such as grid-driven interpolation, in which a single sample point is associated with each grid point would also remove noise coloration, however, reconstruction artifact levels are greater than in regridding, and no sample averaging is included to alleviate motion artifacts.

References: [1] Pipe. MRM (42), 5, 963-969, 1999. [2] Liu, et al. MRM (52), 6, 1388-1396, 2004. [3] Jackson et al. IEEE Trans Med Imag.,10, 473-478,1991. [4] Rosenfeld. MRM (49),1,193-202,2002. **Acknowledgements:** This work was supported in part by the NIH (1R01EB002771), the Center of Advanced MR Technology at Stanford (P41RR09784), Lucas Foundation, and Oak Foundation.