Method of Measuring Diffusion Anisotropy and Diffusion Gradient Simultaneously with DTI

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Introduction: Water diffusion in tissues is generally inhomogeneous and anisotropic due to spatial-location variation (gradient) and spatial-direction variation (anisotropy) of tissue structures. Diffusion anisotropy is measured by DTI [1]. A newly derived theoretical relationship between diffusion gradient and the echo signal in a diffusion-weighted spin-echo pulse sequence establishes the theoretical basis for measuring the diffusion gradient [2]. This paper presents a method of simultaneously measuring diffusion anisotropy and diffusion gradient.

Method and Discussion: For the typical diffusion-weighted spin-echo pulse sequence [1] we write the general solution of the modified Bloch-Torrey equation relating the MR signal at the echo time TE [2]:

$$m(\vec{G}) = m(0)\exp\left(i\vec{d}\cdot\vec{K} - \overleftrightarrow{b} : \overleftrightarrow{D}\right) \quad (1)$$

where $m(\vec{G})$ is the echo MR signal in the presence of the diffusion-encoding magnetic field gradient $\vec{G}$; $m(0)$, the echo MR signal without diffusion-weighting ($\vec{G}$=0); $i \equiv \sqrt{-1}$; $\vec{d} \equiv \gamma\delta\Delta\vec{G}$; $\overleftrightarrow{D}$, the diffusion tensor; $\vec{K} \equiv \nabla\cdot\overleftrightarrow{D}$; and $\overleftrightarrow{b} = \gamma^2\left[\delta^2(\Delta - \delta/3) + \varepsilon^3/30 - \delta\varepsilon^2/6\right]\vec{G}\vec{G}$, the familiar b-factor. As can be seen, $\overleftrightarrow{D}$ attenuates the signal intensity, but $\vec{K}$ causes a phase shift in the signal. Let A and ϕ be the intensity and phase of the signal, *i.e.*, $m = Ae^{-\phi}$, respectively, we found that

$$Ln[A(\vec{G})/A(0)] = -\sum_{i,j} b_{ij}D_{ji} \quad (2a) \quad and \quad \phi(\vec{G}) - \phi(0) = \sum_i d_i K_i \quad (2b)$$

where $A(\vec{G}) = \sqrt{\mathrm{Re}(m(\vec{G}))^2 + \mathrm{Im}(m(\vec{G}))^2}$, $A(0) = \sqrt{\mathrm{Re}(m(0))^2 + \mathrm{Im}(m(0))^2}$, $\phi(\vec{G}) = \tan^{-1}[\mathrm{Im}(m(\vec{G}))/\mathrm{Re}(m(\vec{G}))]$, and $\phi(0) = \tan^{-1}[\mathrm{Im}(m(0))/\mathrm{Re}(m(0))]$. Here, Re and Im represent the real and imaginary part of the signal, respectively. Eqs. (2a) and (2b) show that $\overleftrightarrow{D}$ is determined from the signal intensity, but $\vec{K}$ from its phase. Since an MRI scan is capable of providing both intensity and phase images, Eqs. (2a) and (2b) show that both $\overleftrightarrow{D}$ and $\vec{K}$ can be determined simultaneously, *i.e.*, no additional scans are needed for determining $\vec{K}$. The six independent components of $\overleftrightarrow{D}$ in Eq. (2a) and three components of $\vec{K}$ in Eq. (2b) can be simultaneously determined from the intensities and phases of diffusion-weighted images encoded in six non-collinear directions of the applied diffusion-encoding magnetic field gradients, along with an image acquired without diffusion-weighting ($\vec{G}$=0) for $A(0)$ and $\phi(0)$. Let $X = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T$ and $Y = [y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6]^T$ be a 6×1 matrix that stores these six observations of the left side of Eq. (2a) and of Eq. (2b), respectively. We define a 6×1 matrix as $\alpha = [D_{xx} \quad D_{yy} \quad D_{zz} \quad D_{xy} \quad D_{xz} \quad D_{yz}]^T$, which represents the six tensor components in Eq. (2a), and a 3×1 matrix as $\beta = [K_1 \quad K_2 \quad K_3]^T$, which represents the three components of $\vec{K}$ in Eq. (2b), respectively. The method of least-squares regression was used to yield the optimal estimation,

$$\alpha_{opt} = \left(B^T B\right)^{-1} B^T X \quad (3a) \quad and \quad \beta_{opt} = \left(\Sigma^T \Sigma\right)^{-1}\Sigma^T Y \quad (3b)$$

where B is a 6×6 matrix that is computed from the right side of Eq. (2a) and Σ a 6×3 matrix that is computed from the right side of Eq. (2b), respectively.

The physical meaning of $\vec{K}$ is elucidated in its relationship with the spatial derivatives of the three principal diffusivities:

$$\partial\lambda_i/\partial u_i = \sum_j \varepsilon_{ij} K_j \quad (4)$$

where λ_i and ε_{ij} are the eigenvalues and corresponding eigenvectors of the diffusion tensor, respectively. Within the selected voxel in Fig. 1, white matter fibers converge on the left side and diverge on the right side, yielding a horizontal diffusion gradient along the fiber direction. The associated spatial derivative of the main eigenvalue along the fiber direction could determine the converging and diverging side of the fibers, providing additional information for white matter fiber tracting.

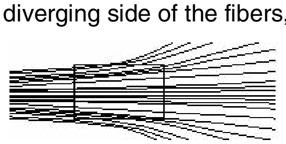

Figure 1. Illustration of converging and diverging white matter fibers with a rectangular box representing an image voxel.

References: 1. Basser PJ, *et al*, J Magn Reson, Series B, **103**: 247-254; 1994. 2. Huang J, Theory of Delineating Tissue Interfaces with Diffusion Gradient-Weighted MRI. Abstract submitted to ISMRM Annual Meeting, 2006.