Evaluation of Cost Functions in the Design of RF Coils Optimized for SENSE Imaging

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Purpose

We have recently proposed a new approach to RF coil array design that is optimized for SENSE imaging by calculating the surface current paths on a coil former that maximized the SNR in SENSE images within a volume of interest (VOI) [1]. The goal of the study presented here was to assess the effect of different weights on components of the cost function, which was used in the *least squares approximation* of the surface current paths. Recently two studies were published in which SENSE optimized coil arrays were investigated by simulating various known coil topographies [2,3]. However, this approach is limited by the number of coils simulated, and the result may not yield the best possible performance.

Methods

The coil system is made of wires placed on a predefined surface in 3D space, which can be approximated by a surface current density J_s. The surface can be any prescribed shape but a cylindrical surface with 28 cm diameter and height was chosen for head imaging. The surface on which J_s flows can be approximated by a Finite Element Mesh (FEM) consisting of flat triangular elements. J_s in each of these finite elements will generate a magnetic vector potential ***A*** and magnetic flux density ***B***, which can be defined at any point in space in terms of J_s. Then, the total ***B*** field is determined by summing over all the elements of the mesh. Details were given in [1,4] and will be omitted here.

The next step is to find the relationship between the SNR_{sense} and the surface current density J_s. so that SNR_{sense} can be used in the cost function to optimize J_s in the least squares sense. SNR_{sense} and the *g-factor* in the ρ^{th} pixel are given by Eq.1-2, respectively [5]. Here, S is the coil sensitivity matrix, ***R*** is the reduction factor, and Ψ is the receiver noise matrix. When only the sample noise is considered, Ψ is given by (Eq.3), where $B_\gamma(\mathbf{r}_i)$ is the field generated by the γth coil at point $\mathbf{r}_i$. Once $SNR_{sense,\rho}$ is formulated in terms of coil ***B*** fields using Eq (1-3), it is possible to calculate J_s that

$$SNR_{sense,\ \rho} = SNR_{full,\ \rho} / (g_\rho \cdot \sqrt{R}) \quad (1)$$

$$g_\rho = \sqrt{((S^H \cdot \Psi^{-1} \cdot S)^{-1})_{\rho,\ \rho} (S^H \cdot \Psi^{-1} \cdot S)_{\rho,\ \rho}} \quad (2)$$

$$\Psi_{\gamma,\ \gamma'} = \sum_{i=1}^{N} B_\gamma(\mathbf{r}_i) \cdot B_{\gamma'}(\mathbf{r}_i) \quad (3)$$

minimized sum of squared ($1/SNR_{sense,\rho}$) in the VOI using a *least squares procedure*. By using the symmetry of the structure, J_s in only one quadrant was calculated and it is confined to one quadrant by boundary conditions to design a four-coil array. This cost function tends to maximize the average SNR inside the VOI. However, in SENSE imaging SNR_{sense} is spatially varying because of the spatial non-uniformity of the *g-factor*. Therefore, the variance of the SNR_{sense} is also added to the cost function and different weighting factors were assigned to each component to emphasize maximizing average SNR while minimizing its spatial non-uniformity. The modified cost function is given in Eq.4. Here, *w1* and *w2* are the weighting factors. Since the first and second terms in Eq.4 can be of different orders of magnitude, a proper normalization is required. This normalization was done by using the first estimates of the SNR_i from the first iteration. We tested the method with the following three normalized weighting factors: *w1*:*w2* = [0.3:0.7, 0.5:0.5, 0.7:0.3]. For the iterations of each coil calculation with different weighting factor pairs, a rectangular coil was given as the initial condition. The VOI was a 16.7cm long cylinder with 22.4cm diameter.

$$C = w1 \cdot \sum_{i=1}^{N} \left(\frac{1}{SNR_i} \right)^2 + w2 \cdot var(SNR) \quad (4)$$

Results and Discussion

Table 1 summarizes the results of simulations with three different combinations of weighting factors. For example, the first row shows results with *w1*:*w2* = 0.3:0.7, which gives more emphasis on variance of SNR. It can be seen from the table that the weighting factor pairs in the first two rows yielded the best results with a trade-off between the higher mean SNR and SNR uniformity. Interestingly, when the weighting factor pair *w1*:*w2* = 0.7:0.3 was used, the mean SNR had more emphasis but yielded the lowest mean SNR while uniformity was significantly compromised. It may be concluded that the variance of SNR, which is closely related to the variance of *g-factor* has a big impact on the cost function. Different weighting pairs will alter the geometry of the cost function, resulting in a variety of local and global minima. In certain cases, the algorithm may converge to a local minimum or the global minimum may be yielding such unexpected results. This will be investigated further. In Fig.1, 3D mesh plots of *g-factors* for the two cases *w1*:*w2* = 0.7:0.3 and *w1*:*w2* = 0.3:0.7 were illustrated. It can be clearly seen that when the weighting of SNR variance was reduced, *g-factor* had sharp peaks, which were suppressed when SNR variance had more weighting. In this study, we have demonstrated that various criteria can be added to the cost function that is used in the least squares approximation of the surface current density for optimum coil design for SENSE imaging. The weighting of these factors has a big impact on the overall mean, as well as the uniformity of SNR. In addition to the criteria for best SNR performance, we have observed that some combinations of weighting factors yielded current paths that are not practical to implement.

Weighting *w1*: *w2*	Mean SNR_{SENSE}	var SNR (norm)	Mean g	Max g
0.3 : 0.7	84.55	1.27	3.46	14.9
0.5 : 0.5	73.22	1.04	3.86	16.7
0.7 : 0.3	72.9	2.3	5.9	61.6

Table 1. Comparison of SNR and g-factor of the SENSE coil array design with different weighting factors of the cost function components. Var(SNR) is normalized by the mean.

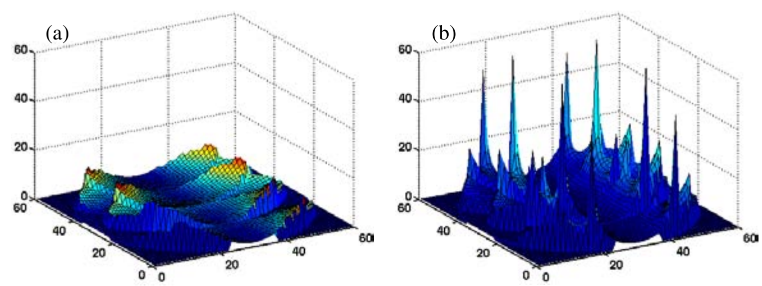

Fig.1. 3D Mesh plots of *g-factor* for the cases of: (a) *w1* :*w2* = 0.3 : 0.7; (b) *w1* :*w2* = 0.7 : 0.3. Note the increased variance due to sharp peaks in (b).

References:

[1] Muftuler et al, Proc. ISMRM 2005, p 886; [2] Weiger M et al, MRM 45: 495-504, 2001; [3] De Zwart JA, et al, MRM 47:1218-1227, 2002; [4] Pissanetzky, Meas.Sci. Technol. 3:667-673, 1992; [5] Pruessmann et al, MRM 42:952-62, 1999