A Novel Fractal Model to Explain the Rheology of Liver Tissue using MR-Elastography

B. Robert[1], R. Sinkus[1], J. Bercoff[2], M. Tanter[1], M. Fink[1]
[1]Laboratoire Ondes et Acoustique, Paris, France, [2]SuperSonic Imagine, Aix-en-Provence, France

Introduction

MR-Elastography (MRE) enables additional characterization of pathologies by providing new physical parameters, for instance the complex shear modulus $G^*(\omega)=G_d(\omega)+G_l(\omega)$ at a single frequency [1,2]. However, interpretation of $G^*(\omega)$ in terms of elastic and viscose properties necessitates the knowledge about the underlying rheological model. Tissue is a material with a hierarchical organization [3] and does not follow for instance the classical Voigt model [4], which is often used to interpret the observed frequency dependence of the measured shear wave speed [5]. Meaningful diagnostic interpretation of $G^*(\omega)$ thus requires studying the underlying rheological model of normal and of pathological tissue in particular. Therefore, as a first step, a study of the frequency behavior of $G^*(\omega)$ of fresh bovine liver tissue is conducted.

Methods

3D Mono-chromatic steady-state MRE experiments of fresh bovine liver samples were performed using mechanical excitation frequencies between 40-100 Hz. $G_d(\omega)$ and $G_l(\omega)$ are reconstructed according to [6] and the obtained maps were averaged spatially. The corresponding mean values for the various frequencies were fit simultaneously to various rheological models. Classical viscoelastic models (Voigt, Maxwell, Zener) are arrangements of finite numbers of springs and dashpots. None of those models is capable to model a power-law, i.e. $G^*(\omega)\sim\omega^\alpha$. This can be accomplished by utilization of a so-called springpot which is mathematically linked to fractional derivatives. These, likewise, are a mathematical necessity if the creep function of the material obeys a power-law [7]. Its rheological interpretation is an infinite series of Maxwell-elements entangled in a fractional manner.

Results

Fig.1 shows the obtained values for $G_d(\omega)$ (red markers) and $G_l(\omega)$ (green markers) as well as $|G^*(\omega)|$ (black markers) together with the best fit of our new model (according lines). The validity of the classical Voigt model is ruled out due to the fact that $G_d(\omega)$ is not constant. Moreover, both datasets rise according to a power-law suggesting a broad distribution of intrinsic relaxation times. Thus, all standard models (Maxwell, Zener) are also excluded because they do not model a power-law behavior. Both individual datasets could well be described by a power-law with $G_d(\omega)\sim G_l(\omega)\sim\omega^{0.75}$. This suggests the validity of the springpot model, whose rheological interpretation is sketched in Fig.2a,b). The model, however, links the ratio of both moduli to the power-law exponent α, i.e. $G_l(\omega)/G_d(\omega) = \tan(\alpha*\pi/2)$ = const. This property resembles so-called structural damping, i.e. elasticity and viscosity originate from the same physical cause. Our data also show a constant ratio. Still, its ratio is not linked to α=0.75 but rather approximated by $G_l(\omega)/G_d(\omega) = \tan(0.15*\pi/2)$. Both observations, power-law behavior $\sim\omega^\alpha$ and constant ratio $G_l(\omega)/G_d(\omega) = \tan(\beta*\pi/2)$, can be explained by a new rheological model which is shown in Fig.2c). Here, a fractal ladder is constructed from springpots as basic elements (unlike the springpot itself, who is a fractal ladder constructed from classical springs and dashpots). This model reflects the properties of two fractal structures, one described by a power-law with exponent α and a second with exponent β. Measurements in phantoms did not necessitate a second network for explanation of the dispersion of $G^*(\omega)$.

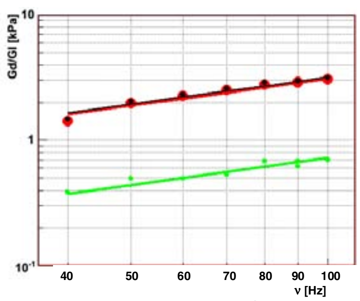

Fig.1: Dynamic modulus G_d (red) and loss modulus G_l (green) as a function of frequency. Black symbols = $|G^*(\omega)|$.

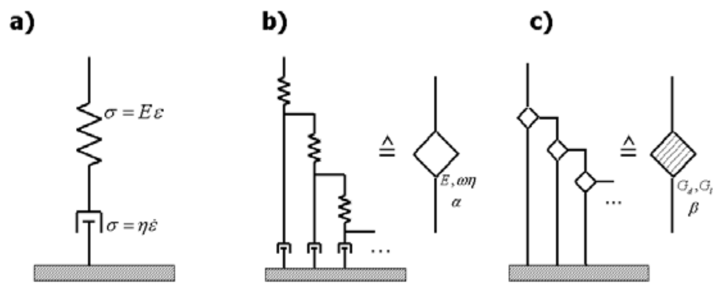

Fig.2: a) Classical Maxwell element consisting of spring and dashpot. b) Infinite fractal ladder constructed from Maxwell elements. This ladder describes a springpot. c) New rheological model which is a fractal ladder of springpots.

Discussion & Conclusions

A proper understanding of the underlying rheology of tissue is required for a reasonable interpretation of clinical data from MRE. This analysis demonstrates the failure of the classical Voigt model to describe the frequency behavior of the dynamic modulus for ex-vivo bovine liver tissue in the frequency range from 40-100 Hz. The observed scaling behavior requires utilization of fractional derivatives which resemble fractal structures. Our data require the extension of the standard springpot model to two fractal networks woven into each other. The inner network yields a power of $\alpha\sim0.75$, which is the value found for the cytoplasm (liquid-like). The outer network yields a power of $\beta\sim0.15$, which has been found for the cytoskeleton (solid-like) [8]. Thus, the explanation of the frequency behavior necessitates utilization of two very different components. Currently, in-vivo rat experiments are conducted to proof which tissue components represent the found power-law exponents. If truly linked to the cytoplasm and the cytoskeleton, MRE might have the capability to reveal micro-rheological properties on the clinical imaging scale due to the intrinsic characteristics of tissue to exhibit scaling.

References

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