Correction on FID NMR signal induced by Mesoscopic Magnetic Field Inhomogeneities at High Volume Fraction

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Introduction

An analytical theory of FID signal formation in the presence of tissue specific magnetic susceptibility effects was developed previously [1]. The theory predicts signal dependence on the susceptibility difference between tissue and magnetized objects, and also on the objects' volume fraction. Theory assumes a sufficient small volume fraction so that a random and independent distribution of the objects can be applied. For spherical objects of radius *R*, the position overlapping occurs if the distance between two objects' centers becomes smaller than 2*R*. Under the volume fraction of 6%, the probability of overlapping of two spheres is 48% whereas the probability of overlapping three or more sphere is less than 1%. This indicates that the overlapping effects should be taken into consideration to adequately describe the signal behavior even for small volume fractions. One such approach (Model of mutually avoiding cylinders) was proposed previously [2]. Herein we developed an approach that combines mean field approximation with the hard-sphere model [3, 4] to estimate the correction on NMR signal. Under such approximation, the distribution function of a given sphere is formulated under the influence of the rest of objects.

Theory

Let us consider *N* spherical objects of radius r_0 and volume v_0 embedded in the given medium with the volume fraction $\zeta = N\, v_0 / V$. The FID signal after an RF excitation pulse, normalized to the system volume *V*, can be represented as

$$\langle S(t)\rangle = \frac{1}{V}\cdot\int_V d\mathbf{r}\int_V d\mathbf{r}_1\int_V d\mathbf{r}_2\cdots\int_V d\mathbf{r}_N\cdot P(\mathbf{r};\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_N)\cdot\prod_{n=1}^{N}\exp\left[-i\cdot\gamma\cdot b_n(\mathbf{r}-\mathbf{r}_n)\cdot t\right] \quad (1)$$

where $b_n(\mathbf{r}-\mathbf{r}_n)$ is the *z* component of the inhomogeneous magnetic field induced by the n^{th} object (with the center at $\mathbf{r}_n$) at a point $\mathbf{r}$; $P(.)$ is the probability to find a spin at a point $\mathbf{r}$ and the centers of objects at $\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N$. Under the hard-sphere model, the distribution functions of the objects are not independent from each other. However, in the mean field approximation, each object can still be approximately treated independently with a modified distribution function. This distribution function can be formulated in an imaginary process in which the specific object is introduced from the outside of system, which is pre-packed with (*N* -1) hard-sphere objects.

Given a medium spin at **r** and the n^{th} object located at $\mathbf{r}_n$, then for each of the rest of (*N* -1) objects, its distance to **r** should be larger than r_0 and its distance to $\mathbf{r}_n$ should be larger than $2r_0$. The "forbidden" region is called as an exclusive region. The volume of the exclusive region (denoted hereafter $\lambda(R_n)\cdot v_0$) depends on $R_n = |\mathbf{r}_n - \mathbf{r}|$: for example, $\lambda = 9$ for $R_n > 3r_0$, $\lambda = 8$ for $R_n < r_0$. Assuming the rest of (*N*–1) objects are distributed independently and uniformly, the probability of finding that none of the (*N*-1) object resides in this exclusive area is $(1-\lambda\cdot v_0/V)^{N-1}\approx\exp(-\lambda\cdot\zeta)$. Since the existence of any of (*N*-1) object inside this exclusive region would automatically forbid the allocation of n^{th} object at the point $\mathbf{r}_n$, the distribution function for the n^{th} object is not uniform (outside of the exclusive region created by the spin at **r**) anymore. The tedious calculations give the following result: the distribution function for the n^{th} object is 0 if $R_n < r_0$; $\exp\left[\left(9-\lambda\left(R_n/r_0\right)\right)\cdot\zeta\right]/\Omega$ if $r_0 < R_n < 3r_0$; and $1/\Omega$ if $R_n > 3r_0$; where Ω is the normalization factor. It means that the n^{th} object has a higher probability to stay closer to the given spin at **r** since the exclusive region becomes smaller. Then the FID signal in Eq. (1) becomes

$$\langle S(t)\rangle = (1-\zeta)\cdot\exp\left[-\varsigma\cdot\left(f_s(\delta\omega_s\cdot t)+\Delta f_s(\delta\omega_s\cdot t)\right)\right],\quad \Delta f_s(\delta\omega_s\cdot t)=\int_0^1 dU\int_{1/27}^1 du\cdot\frac{1-e^{-i\cdot\delta\omega_s\cdot t\cdot u\cdot(3U^2-1)}}{u^2}\cdot\left(e^{\left(9-\lambda\left(u^{-1/3}\right)\right)\cdot\zeta}-1\right) \quad (2)$$

where $f_s(.)$ is the characteristic function for spherical object described in [1]. In the short time regime, when $\delta\omega_s\cdot t \ll 1$,

$$\mathrm{Re}\left(f_s(\varphi)\right)\approx 0.4\cdot\varphi^2+O(\varphi^3),\qquad \mathrm{Re}\left(\Delta f_s(\varphi)\right)\approx 0.4\cdot\zeta\cdot\varphi^2\cdot\left[0.79+O(\zeta)\right] \quad (3)$$

In the long time regime, when $\delta\omega_s\cdot t \gg 1$,

$$\mathrm{Re}\left(f_s(\varphi)\right)\approx 1.21\cdot\varphi-1+O(\varphi^{-1}),\qquad \mathrm{Re}\left(\Delta f_s(\varphi)\right)\approx 7\zeta+O(\zeta^2) \quad (4)$$

Discussion

Under the solid-sphere model, the distribution function of a magnetized object around a fixed spin is not uniform due to the position dependency of the exclusive region. As a result, in the hard-sphere model a spin experiences a higher susceptibility induced magnetic field inhomogeneities than in the model permitting object overlapping. Higher mesoscopic field inhomogeneities lead to more rapid decay of the FID signal.

In the short-time regime, Eq.(3), the correction term in the signal decay rate is quadratic as in [1] with modified coefficient. In the long time regime, Eq. (4), the correction term contributes an extra time-independent constant term. Combined with $f_s(.)$, the modified function will have a constant term of ($7\cdot\zeta-1$) instead of -1. As the volume fraction can be determined by extrapolating the linear time dependence of ln*S* at $t \gg 1/\delta\omega_s$ to t = 0 [5], the correction term 7ζ should be taken into account for correct measurements. For example, in the case of a real volume fraction of 6%, its estimated value without the correction term would be only 3.12%. Even for the case of a small volume fraction of 3%, it will be underestimated by 20% if the correction term is ignored.

Reference

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