Bayesian Parallel Imaging with Edge-Preserving Priors

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INTRODUCTION

Existing parallel imaging methods are limited by a fundamental tradeoff, where suppressing background noise introduces aliasing artifacts. Reconstructed data is further degraded by sensitivity errors due to physiological motion, coil misalignment and insufficient calibration lines. Current regularization techniques either impose minimum norm (1), or require a prior image mean estimate. The limitations of these regularization techniques are clear: regularized SENSE makes unrealistic assumptions about the image norm, while methods relying on a prior estimate (2) must be carefully registered to the target. Use of such strong reference priors is vulnerable to errors in their estimation, leading to reconstruction artifacts. Tikhonov regularization (2) incorporates spatial information, but unfortunately assume that intensities are globally smooth, leading to excessive blurring of edges. We introduce an edge-preserving prior that instead assumes that intensities are piecewise smooth, and show how to efficiently compute its Bayesian estimate. Our prior model is quite general, and has very few parameters; hence little or no effort is required to find this prior, in contrast to image-based or temporal priors. The estimation task is formulated as an optimization problem, which requires minimizing a non-convex objective function in a space with thousands of dimensions. As a result, traditional continuous minimization methods cannot be applied. However, our optimization problem is closely related to some problems in computer vision for which discrete optimization methods based on graph cuts have been developed in the last few years. We extend these algorithms to address our optimization problem, and call it EPIGRAM (Edge-Preserving Parallel Imaging with Graph Cut Minimization).

METHOD

For cartesian parallel imaging with acceleration by R, the N x M image X folds over into N/R x M aliased coil outputs Y_l, and has a linear form $\mathbf{y} = \mathbf{Ex}$ for vectors $\mathbf{x}$ and $\mathbf{y}$, and matrix $\mathbf{E}$. We seek to minimize $E(\mathbf{x}) = \|\mathbf{y} - \mathbf{E}\mathbf{x}\|^2 + G_{EP}(\mathbf{x})$, where we introduce $G_{EP}(\mathbf{x}) = \sum_{(p,q)\in N_s} V(x_p, x_q)$, $V(x_p, x_q) = \min(|x_p - x_q|, K)$, a new class of edge-preserving smoothness penalties. The neighbourhood system N_s consists of pairs of adjacent pixels p and q. Neighbouring intensity differences within the threshold K are treated as noise and penalized accordingly by $V(x_p, x_q)$, but larger differences are not further penalized since they most likely occur at edges. This is very different from traditional convex L_2 separation cost used in (1,2) which effectively forbids two adjacent pixels from having very different intensities. The aliasing pixels in X that contribute to pixel $\overline{p} = (i, j)$ in Y_l are defined by the aliasing set $\mathrm{N}_a = \{(p, p'),\ [p] = [p']\}$, where $[p] = (mod(i, N/R), j)$. It can be shown (3) that

$$E(\mathbf{x}) = a^2 + \sum_p b(p)X^2(p) - 2\sum_p c(p)X(p) + 2\sum_{(p,p')\in \mathrm{N}_a} d(p,p')X(p)X(p') + \sum_{(p,q)\in N_s} V(x_p, x_q) \qquad [1]$$

for appropriately chosen functions $b(p)$, $c(p)$ and $d(p,p')$. Because of highly non-convex functions $V(x_p, x_q)$, [1] cannot be easily minimized using traditional methods like conjugate gradients. Eq. [1] is in a form that was efficiently minimized in computer vision using a powerful discrete optimization technique called graph cuts (4). This technique converts the minimization [1] into a series of binary minimizations involving only a single intensity at a time. Thus, suppose X contains discrete intensities in the range [0,255]. The approach is to pick one value, say 0, and assign it to pixels in a way that minimizes [1]. Repeat this process for all intensities, and loop until [1] can no longer be reduced. These successive minimizations are efficiently performed by graph cuts by constructing a graph whose nodes correspond to image voxels and whose edges correspond to individual terms in the cost function [1]. Unfortunately, graph cuts can minimize functions like [1] only in the absence of cross terms involving $d(p,p')$; for our problem, we need to extend traditional graph cut algorithms. We do this by constructing an augmented graph where instead of representing each voxel by a single node, we use two nodes. It can be shown that this new graph represents an objective function which is minimizable by graph cuts.

RESULTS

Fig 1 shows reconstruction of a central sagittal MPRAGE slice using an 8-channel head coil on a 4T Bruker /Siemens machine, with an undersampling factor $R=4$. The sum of squares image is shown in (a), regularized SENSE with (empirically obtained optimal) $\mu = 0.12$ in (b), regularized SENSE with $\mu = 0.24$ in (c) and EPIGRAM in (d). Considerable SNR improvement is observed in the EPIGRAM reconstruction, compared to both SENSE reconstructions. Higher regularization in SENSE caused unacceptable residual aliasing, as observed by (c). Fig 2 shows similar results for an axial torso scan; SENSE was regularized with $\mu = 0.16$ in (b) and (e).

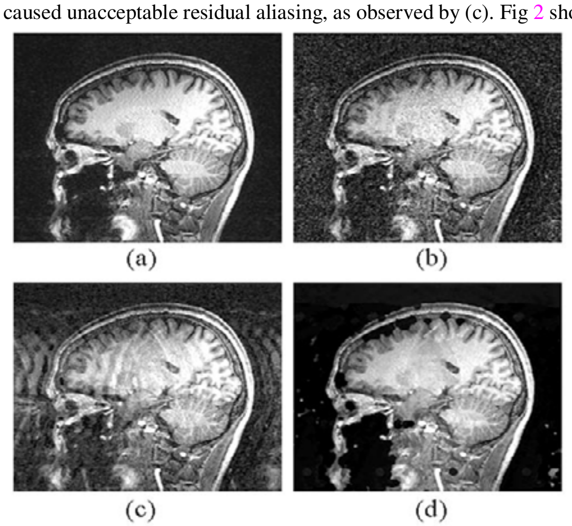

Figure 1

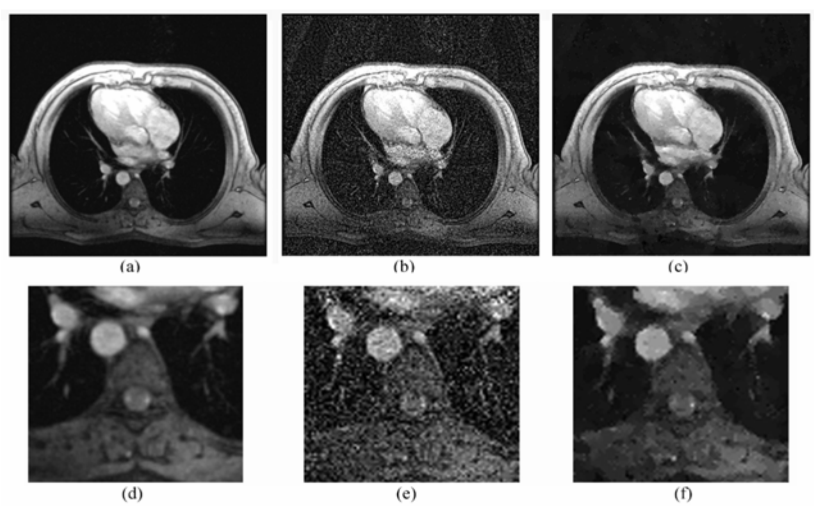

Figure 2

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