# Iterative solution of Transmit SENSE using a conjugate gradient method


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## Introduction

Due to the analogy between the design of spatially selective RF pulses and MR imaging [1], it is possible to adapt the methodology of parallel imaging [2, 3] for parallel transmission [4-6]. Thus, the RF pulse calculation can be performed via direct matrix inversion [4-6], which in parallel imaging has been used by, e.g., [2, 3]. On the other hand, the RF pulse calculation of parallel transmission might be performed via iterative methods [7, 8], which in parallel imaging has been used by, e.g., [9]. This paper discusses the iterative approach in some detail and compares the two approaches with respect to their running times for different scenarios.

## Methods

In analogy with the non-Cartesian SENSE reconstruction [9], the matrix inversion of the RF excitation pulse calculation was implemented iteratively, using a conjugate-gradient (CG) method (see Fig. 1) in the image domain. For this purpose, the algorithm of [9] for receive SENSE was modified by replacing the gridding step with a direct calculation of the convolution in k-space to improve the accuracy of the results.

For the iteration, the linear equation system $\mu(x')g(x') = \sum_l \left[ \mu(x') S_l(x') \sum_x dPSF(x'-x)(S_l^*(x)\mu(x)\breve{g}(x)) \right]$ has to be solved for $\breve{g}(x)$ with $dPSF(x'-x) = \sum_t \frac{e^{2\pi i(x'-x)k(t)}}{D(t)}$ the density compensated point spread function, $g(x) = -i\frac{M_T(x)}{M_0}$ the normalized excitation pattern, $\mu(x)$ the intensity correction, $S_l(x)$ the coil sensitivities, $l$ the coil index, $*$ the conjugate complex, $D(t)$ the density correlation values, $M_T$ the transverse magnetization, and $M_0$ the equilibrium magnetization. Then, the $l$ excitation currents $I(t)$ are obtained by [6] $I_l(t) = \frac{1}{D(t)} \sum_x S_l^*(x)\mu(x)\,\breve{g}(x)\,e^{-2\pi i\,x\,k(t)}$. In case of [6], the matrix inversion is carried out directly, while in [8], a conjugate gradient approach was used.

For non-parallel RF-pulse design, the iterative approach was first presented by [7].

Simulated coil sensitivities from an eight-channel body coil with TEM resonators were used for the calculation. Five different excitation patterns centered in the field of excitation (FOX) were used: (a) circle with a diameter of half the image size, (b) a peak of one pixel width, (c) a ring with a diameter of half the image size, (d) a horizontal slice with a width of a quarter of the image size and (e) a checkerboard. Different matrix sizes $N$ for the FOX discretization were tested ($N$=8×8, 16×16, … 128×128 pixels). For the 2D RF-pulses, a constant angular speed spiral trajectory was used. All pulses were accelerated by a reduction factor $R$=2.

For the performance comparison, all initializations and file I/O were excluded from the measurements. The correlation between the desired and simulated excitation pattern was calculated over the full FOX and exceeded a correlation of 99% in all cases.

All measurements were carried out under Win XP on a Workstation (Dell Precision Workstation 350) with Intel P4 running at 3.06 GHz, equipped with 1 GB / PC533 of memory using a high precision timer.

## Results and Discussion

The performance measurements of both algorithms are summarized in Fig. 2. It shows the running time vs. the matrix size $N$ of the FOX. The running time for the iterative approach crucially depends on the aspired accuracy of the result. If a correlation of 99.0% between desired and simulated pattern is aspired, the iterative approach (solid circles) outperforms the direct calculation (squares) for all matrix sizes $N$. The difference increases from a factor of about 4 for $N = 8^2$ to more than an order of magnitude for $N = 64^2$. However, if a correlation of 99.9% is aspired, which corresponds - for the investigated scenario - to the correlation usually achieved using the matrix inversion, the running time of the iterative approach increases roughly by a factor 5. In this case, the iterative approach outperforms the direct calculation only for matrix sizes above $N = 32^2$.

The running time for the direct calculation is rather independent of the pattern as well as the type of trajectory used. However, for non-Cartesian trajectories, the determination of the regularization parameter to achieve a desired correlation requires carrying out the calculation multiple times. This leads to an additional increase of the running time.

In the iterative case, the running time varies for the different excitation patterns. For the five different excitation patterns used, this variation is indicated by the vertical bars in Fig. 2. Furthermore, the number of iterations increases in order to achieve the same correlation for non-Cartesian compared to Cartesian trajectories.

It is worth mentioning that for the iterative approach a dependency by up to a factor of 5 was found, when using different coil sensitivities.

## Conclusion

The iterative calculation of 2D RF excitation pulses for Transmit SENSE is advantageous compared with the direct matrix inversion in particular for large matrices with respect to the running time.

## References


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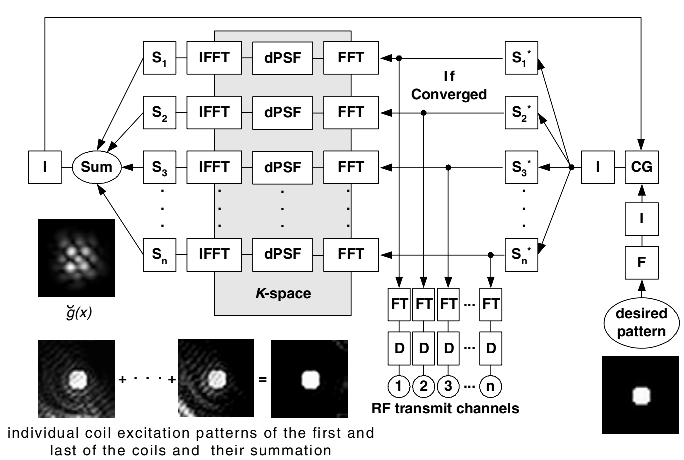


Fig. 1: Algorithm of the iterative calculation of Transmit SENSE pulses in the image domain

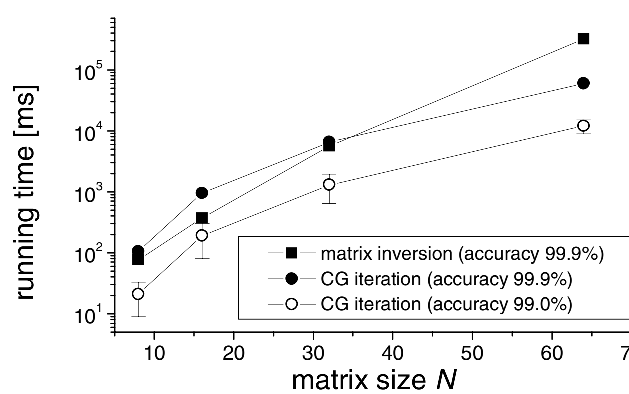


Fig. 2: Performance comparison of different approaches for calculating Tx SENSE pulses. For an excitation accuracy of 99.9%, matrix inversion is faster for small and CG iteration faster for large matrix sizes $N$.